# International Comovement of $r^*$: A Case Study of the G7 Countries

Online Appendix

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*JEL classification:* C32; E43; F44
*Keywords:* The natural rate of interest; G7; State-space model; Decomposition

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### 2.2 Prior Specification for the Country-by-Country Estimation

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### 2.3 Prior Specification for the Stationary Processes on the Preference Shifters

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## 3 A Simple Model Explaining the Link Between Output Growth and Real Interest Rate

Consider an economy where technology $A_t$ and labor $L_t$ grow at rates $g_A$ and $n$. Production uses labor and capital, and the production function is labor-augmenting and constant returns to scale. The representative consumer's optimal intertemporal consumption path or Eular equation is:

$$u^{'}(c_t) = \frac{1}{1+\rho} u^{'}(c_{t+1})(F_K(K_t, A_t L_t) + 1) \tag{1}$$

where $u^{'}(c_t) = \dfrac{du(c_t)}{dc_{c_t}}$, $\dfrac{1}{1+\rho}$ represents time preference with $0 < \dfrac{1}{1+\rho} < 1$, and $F_K(K_t, A_t L_t) = \dfrac{\partial F(K_t, A_t L_t)}{\partial K_t} = r_t$.

Here, the utility function takes a form of constant relative risk aversion (CRRA):

$$u(c_t) = \frac{c_t^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} \tag{2}$$

Then equation (1) becomes:

$$\left(\frac{c_{t+1}}{c_t}\right)^{\frac{1}{\sigma}} = \frac{1}{1+\rho}(r_t + 1) \tag{3}$$

Take logarithm on both sides:

$$\frac{1}{\sigma}\ln\left(\frac{c_{t+1}}{c_t}\right) = \ln(r_t + 1) - \ln(1+\rho)$$

Using the first order Taylor approximation:

$$\frac{1}{\sigma}\left(\frac{c_{t+1} - c_t}{c_t}\right) \approx r_t - \rho \tag{4}$$

using the fact that $\ln\left(\dfrac{x_{t+1}}{x_t}\right) \approx \dfrac{x_{t+1} - x_t}{x_t}$ and $\ln(x_t + 1) \approx x_t$.

Under a balanced growth path and from equation (4), we obtain:

$$r^* = \sigma^{-1} g_A + \rho$$

This is the equation in Holston et al. (2017) that summarizes the one-to-one relationship of output growth and real interest rate.

# 4 A Complete Description of the State Space Model

#### 4.0.1 Measurement Equations

## 4.1 Measurement Equations

$$Y_t = AX_t + HM_t + \nu_t$$

or

$$\begin{bmatrix} Y_{CA,t} \\ \vdots \\ Y_{US,t} \end{bmatrix} = \begin{bmatrix} A_{CA} & & \text{\huge 0} \\ & \ddots & \\ \text{\huge 0} & & A_{US} \end{bmatrix} \begin{bmatrix} x_{CA,t} \\ \vdots \\ x_{US,t} \end{bmatrix} + \begin{bmatrix} H_{CA} & & \text{\huge 0} & -\frac{a_{r,CA}}{2}_{1\times 4} & \\ & \ddots & & \vdots & \text{\huge h} \\ \text{\huge 0} & & H_{US} & -\frac{a_{r,US}}{2}_{1\times 4} & \end{bmatrix} M_t + \begin{bmatrix} \nu_{CA,t} \\ \vdots \\ \nu_{US,t} \end{bmatrix}$$

where

$$Y_{i,t} = \begin{bmatrix} y_{i,t} \\ \pi_{i,t} \end{bmatrix}, \quad A_i = \begin{bmatrix} a_{y1,i} & a_{y2,i} & \frac{a_{r,i}}{2} & \frac{a_{r,i}}{2} & 0 & 0 \\ b_{y,i} & 0 & 0 & 0 & b_{\pi,i} & 1 - b_{\pi,i} \end{bmatrix}, \quad x_{i,t} = \begin{bmatrix} y_{i,t-1} \\ y_{i,t-1} \\ r_{i,t-1} \\ r_{i,t-2} \\ \pi_{i,t-1} \\ \pi_{i,t-2,4} \end{bmatrix}$$

$$H_i = \begin{bmatrix} 1 & -a_{y1,i} & -a_{y2,i} & -\frac{a_{r,i}}{2} & -\frac{a_{r,i}}{2} & -\frac{a_{r,i}}{2} & -\frac{a_{r,i}}{2} \\ 0 & -b_{y,i} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathrm{h} = \begin{bmatrix} -\frac{a_{r,CA}}{2}{}_{1\times 4} & 0_{1\times 4} \\ 0_{1\times 4} & 0_{1\times 4} \\ 0_{1\times 4} & -\frac{a_{r,FR}}{2}{}_{1\times 4} \\ 0_{1\times 4} & 0_{1\times 4} \\ 0_{1\times 4} & -\frac{a_{r,DE}}{2}{}_{1\times 4} \\ 0_{1\times 4} & 0_{1\times 4} \\ 0_{1\times 4} & -\frac{a_{r,IT}}{2}{}_{1\times 4} \\ 0_{1\times 4} & 0_{1\times 4} \\ 0_{1\times 4} & 0_{1\times 4} \\ 0_{1\times 4} & 0_{1\times 4} \\ 0_{1\times 4} & -\frac{a_{r,UK}}{2}{}_{1\times 4} \\ 0_{1\times 4} & 0_{1\times 4} \\ -\frac{a_{r,US}}{2}{}_{1\times 4} & 0_{1\times 4} \\ 0_{1\times 4} & 0_{1\times 4} \end{bmatrix},$$

$$Mt = \begin{bmatrix} \xi_{CA,t} \\ \vdots \\ \xi_{US,t} \\ g_{t-1}^{common} \\ g_{t-2}^{common} \\ z_{t-1}^{common} \\ z_{t-2}^{common} \\ g_{t-1}^{north-america} \\ g_{t-2}^{north-america} \\ z_{t-1}^{north-america} \\ z_{t-2}^{north-america} \\ g_{t-1}^{europe} \\ g_{t-2}^{europe} \\ z_{t-1}^{europe} \\ z_{t-2}^{europe} \end{bmatrix}$$

$$\text{with} \quad \xi_{i,t} = \begin{bmatrix} y_{i,t}^{*} \\ y_{i,t-1}^{*} \\ y_{i,t-2}^{*} \\ g_{t-1}^{idiosyncratic} \\ g_{t-2}^{idiosyncratic} \\ z_{t-1}^{idiosyncratic} \\ z_{t-2}^{idiosyncratic} \end{bmatrix}$$

### 4.1.1 State Equations

$$M_t = FM_{t-1} + \epsilon_t$$

or

$$M_t = \begin{bmatrix} f & & \text{\Large 0} & \omega & z_{CA} \\ & \ddots & & \vdots & \vdots \\ \text{\Large 0} & & f & \omega & z_{US} \\ \mathbf{0} & \dots & \mathbf{0} & & \delta \end{bmatrix} M_{t-1} + \begin{bmatrix} \epsilon_{CA,t} \\ \vdots \\ \epsilon_{US,t} \\ e_t \end{bmatrix}$$

where

$$f = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \omega = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad z_{CA} = z_{US} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$z_{FR} = z_{DE} = z_{IT} = z_{US} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad z_{JP} = \mathbf{0}_{7\times 8}$$

$$\delta = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&1&0 \end{bmatrix}$$

$$\epsilon_{i,t} = \begin{bmatrix} \epsilon_{t,y_i^*} \\ 0 \\ 0 \\ \epsilon_{t,g_i^{idiosyncratic}} \\ 0 \\ \epsilon_{t,z_i^{idiosyncratic}} \\ 0 \end{bmatrix}, \quad \text{and} \quad e_t = \begin{bmatrix} \epsilon_{t,g^{common}} \\ 0 \\ \epsilon_{t,z^{common}} \\ 0 \\ \epsilon_{t,g^{north-america}} \\ 0 \\ \epsilon_{t,z^{north-america}} \\ 0 \\ \epsilon_{t,g^{europe}} \\ 0 \\ \epsilon_{t,z^{europe}} \\ 0 \end{bmatrix}$$

$$\nu_t \sim \mathcal{N}(\mathbf{0}, R) \qquad \text{where} \qquad R = \begin{bmatrix} R_{CA} & & \text{\Large 0} \\ & \ddots & \\ \text{\Large 0} & & R_{US} \end{bmatrix} \quad \text{with} \quad R_i = \begin{bmatrix} \sigma_{\tilde{y}_i} & 0 \\ 0 & \sigma_{\pi i} \end{bmatrix}$$

$$\epsilon_t \sim \mathcal{N}(\mathbf{0}, Q) \qquad \text{where} \qquad Q = \begin{bmatrix} q_{CA} & & \text{\Large 0} & \mathit{\Omega} & s_{CA} \\ & \ddots & & \vdots & \vdots \\ \text{\Large 0} & & q_{US} & \mathit{\Omega} & s_{US} \\ \mathit{\Omega}^T & \dots & \mathit{\Omega}^T & & \\ s_{CA}^T & \dots & s_{US}^T & & \mathit{\Gamma}_{2\times 2} \end{bmatrix}$$

Here

$$q_{CA} = q_{US} = \begin{bmatrix} \sigma^2_{y^*i} + \sigma^2_{g^{common}} + \sigma^2_{g^{north-america}} + \sigma^2_{g_i} & 0 & 0 & \sigma^2_{g_i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma^2_{g_i} & 0 & 0 & \sigma^2_{g_i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2_{z_i} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$q_{FR} = q_{DE} = q_{IT} = q_{UK} = \begin{bmatrix} \sigma^2_{y^*i} + \sigma^2_{g^{common}} + \sigma^2_{g^{europe}} + \sigma^2_{g_i} & 0 & 0 & \sigma^2_{g_i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma^2_{g_i} & 0 & 0 & \sigma^2_{g_i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2_{z_i} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$q_{JP} = \begin{bmatrix} \sigma^2_{y^*JP} + \sigma^2_{g^{common}} + \sigma^2_{g_i} & 0 & 0 & \sigma^2_{g_{JP}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma^2_{g_{JP}} & 0 & 0 & \sigma^2_{g_{JP}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2_{z_{JP}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Omega = \begin{bmatrix} \sigma^2_{g^{common}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad s_{CA} = s_{US} = \begin{bmatrix} \sigma^2_{g^{north-america}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$s_{FR} = s_{DE} = s_{IT} = s_{UK} = \begin{bmatrix} 0 & 0 & 0 & 0 & \sigma^2_{g^{europe}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad s_{JP} = \mathbf{0}_{8\times 8}, \quad \text{and}$$

$$\Gamma = \begin{bmatrix} \sigma^2_{g^{common}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{z^{common}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma^2_{g^{north-america}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma^2_{z^{north-america}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma^2_{g^{europe}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma^2_{z^{europe}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

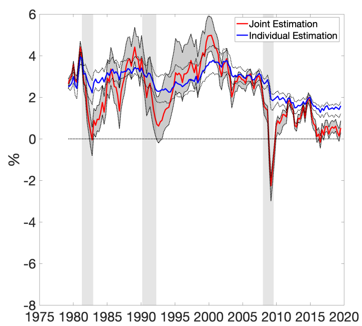
Finally, $\nu_t$ and $\epsilon_t$ follow Gaussian distribution and are mutually uncorrelated.

# References

Holston, K., T. Laubach, and J. C. Williams (2017). Measuring the natural rate of interest: International trends and determinants. *Journal of International Economics 108*, S59–S75.

# List of Figures

Figure 1: The Natural Rate of Interest: Joint Estimates vs Country-by-Country Estimates

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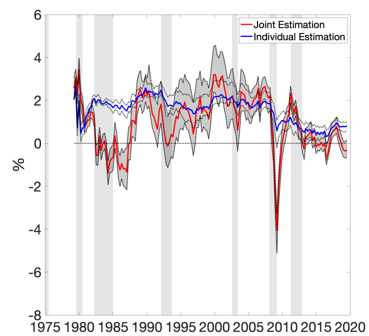


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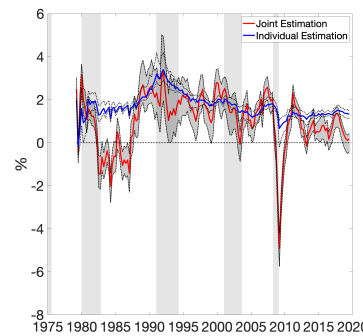
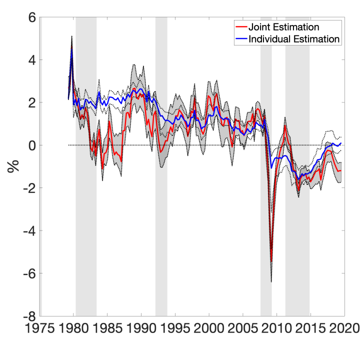



(c) Germany

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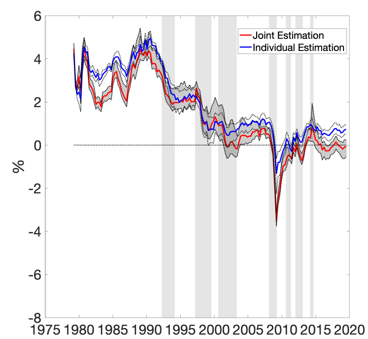


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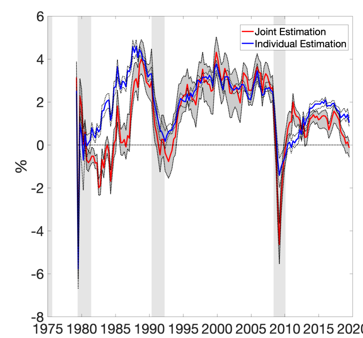
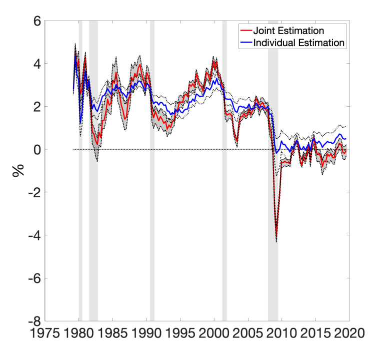
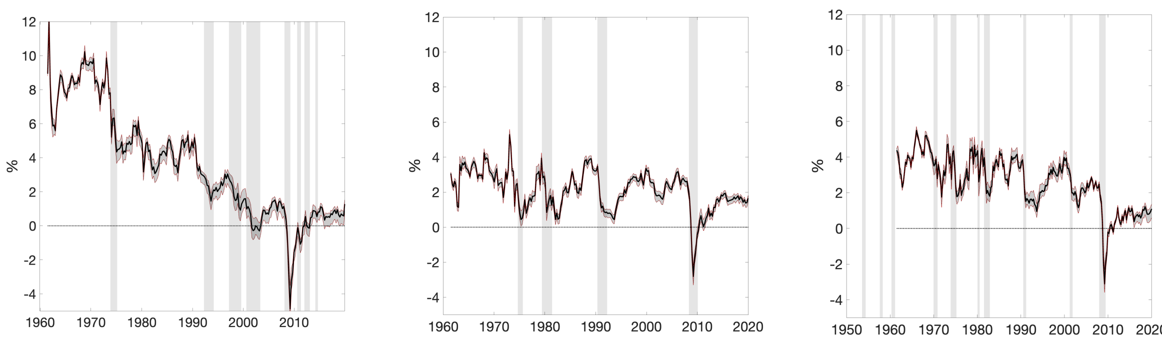



(f) United Kingdom



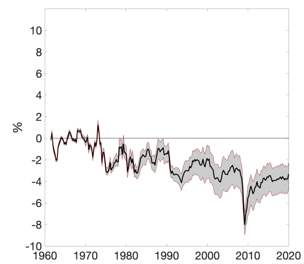

(g) United States

Figure 2: The Natural Rate of Interest (Three Country Estimation)

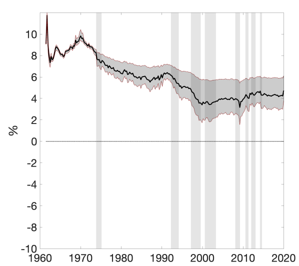



(a) Japan (b) United Kingdom (c) United States

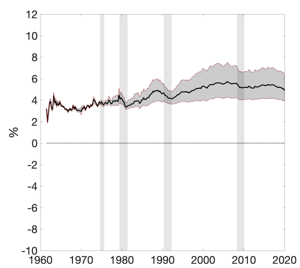
Figure 3: The Natural Rate of Interest Decomposition (Three Country Estimation)



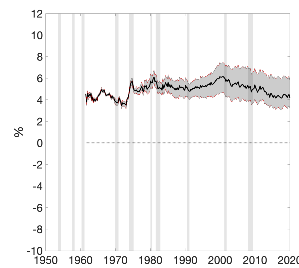

(a) Common Component




(b) Idiosyncratic (Japan)

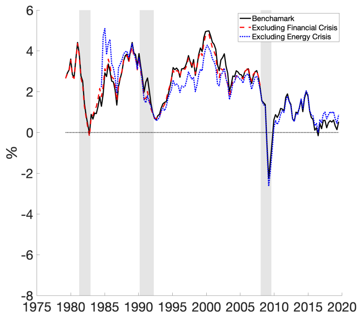



(c) Idiosyncratic (United Kingdom)




(d) Idiosyncratic (United States)

Figure 4: The Natural Rate of Interest (Sub-Sample Estimates)

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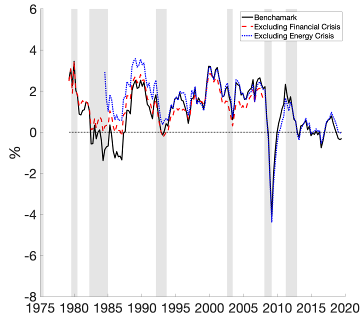
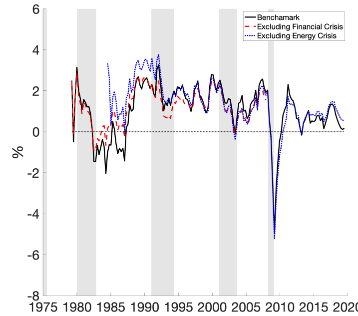
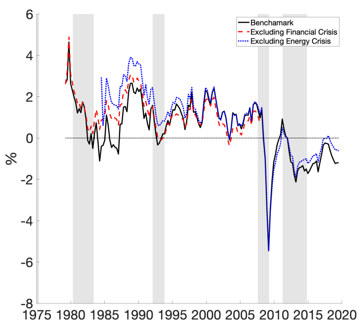


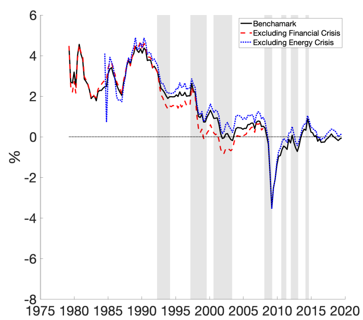

(b) France




(c) Germany




(d) Italy

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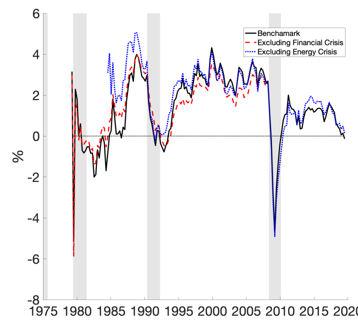


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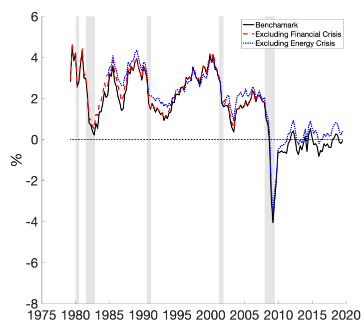
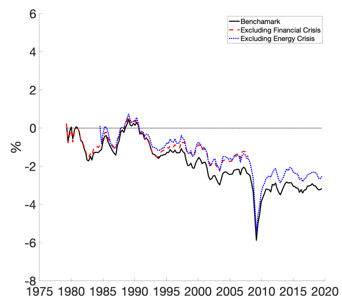
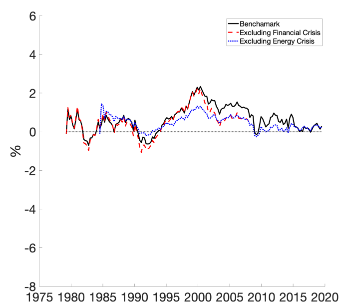



(g) United States

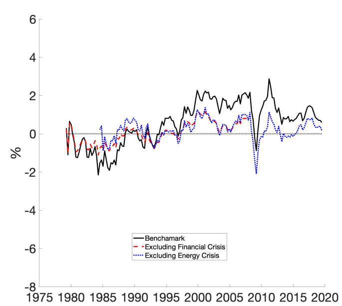
Figure 5: Common Component and Regional Component of $r^*$ (Sub-Sample Estimates)




(a) Common Component

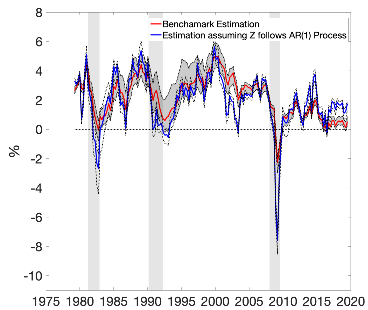



(b) North America Component

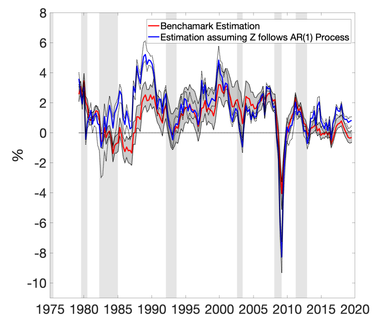



(c) Europe Component

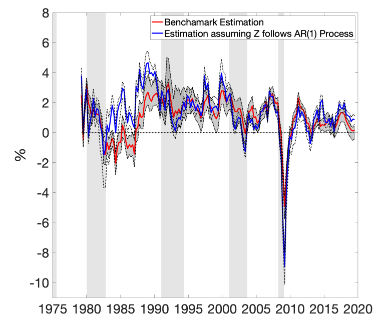
Figure 6: The Natural Rate of Interest (Stationary Processes on the Preference Shifter)




(a) Canada




(b) France

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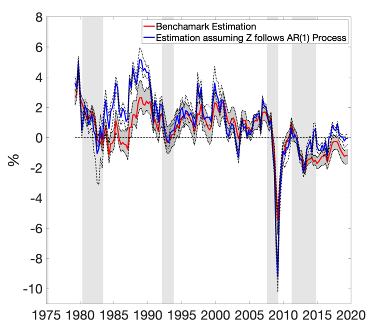
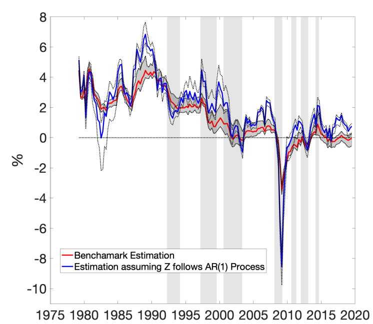
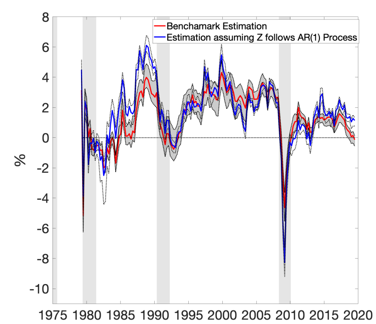


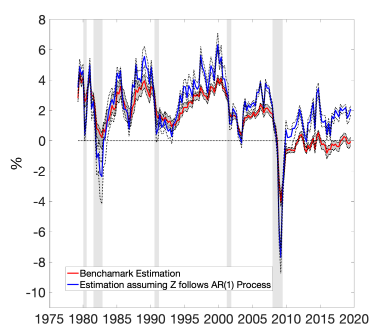

(d) Italy




(e) Japan

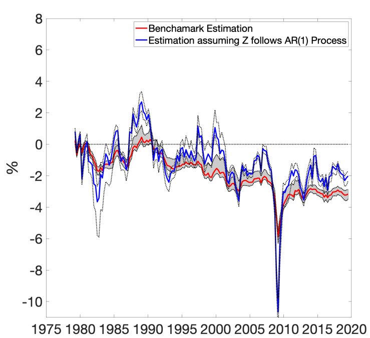



(f) United Kingdom

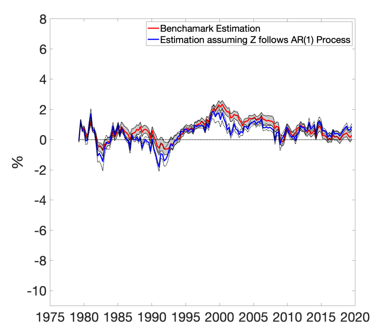



(g) United States

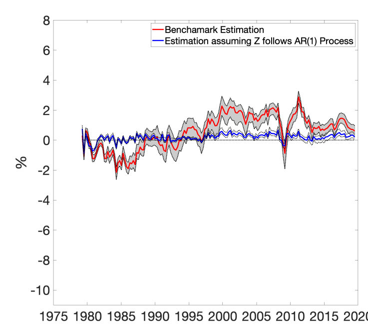
Figure 7: The Natural Rate of Interest Decomposition (Stationary Processes on the Preference Shifter)




(a) Common Component

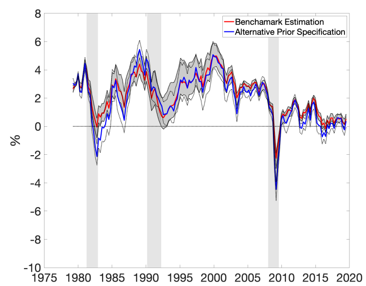



(b) North America Component

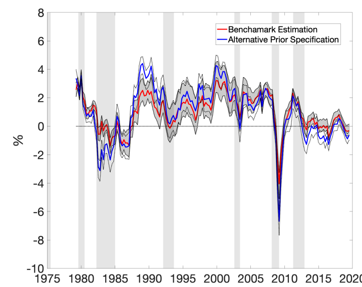



(c) Europe Component

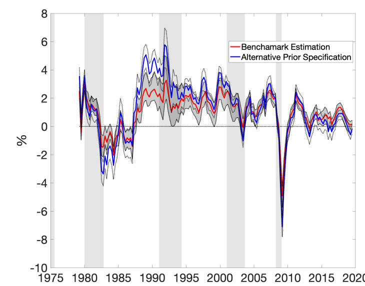
Figure 8: The Natural Rate of Interest (A Diffuse Prior)



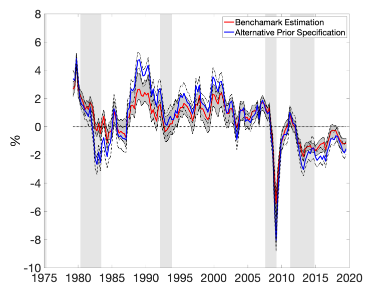

(a) Canada



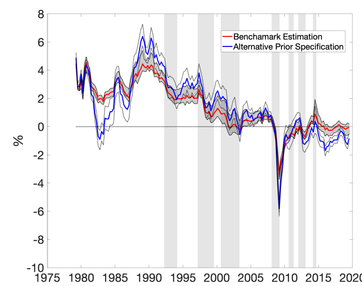

(b) France



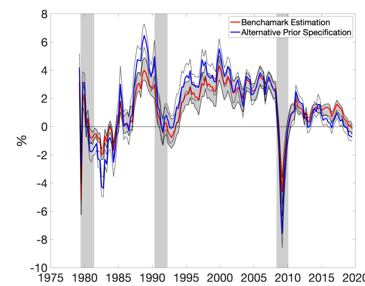

(c) Germany



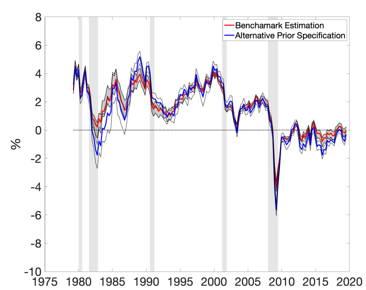

(d) Italy




(e) Japan

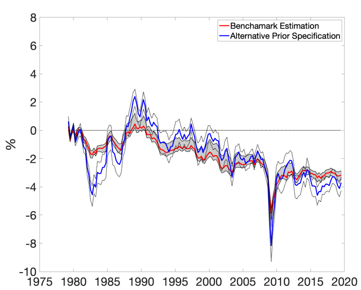



(f) United Kingdom

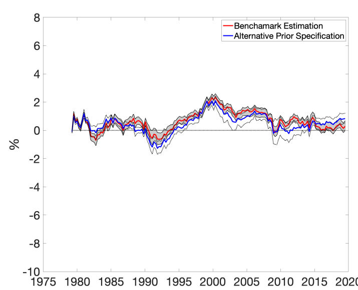



(g) United States

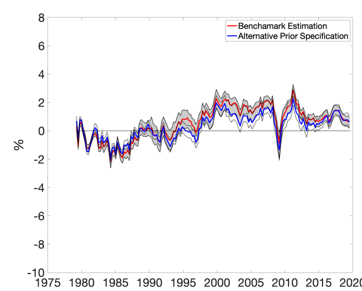
Figure 9: The Natural Rate of Interest Decomposition (A Diffuse Prior)




(a) Common Component

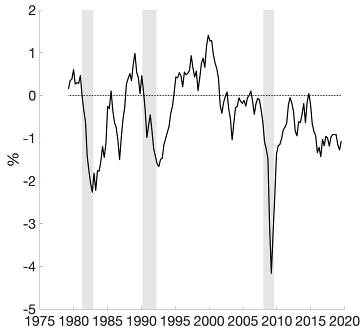



(b) North America Component

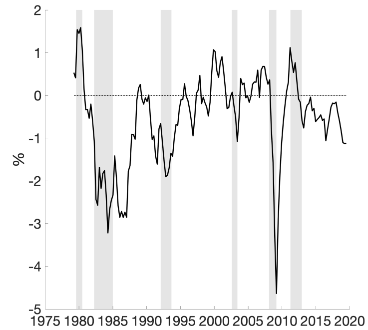



(c) Europe Component

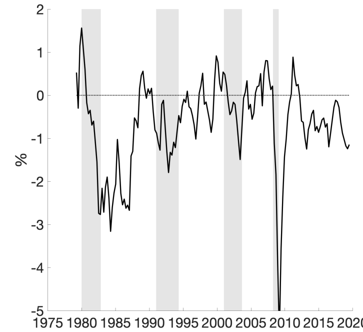
Figure 10: The Natural Rate of Interest: Joint Estimates Minus Country-by-Country Estimates



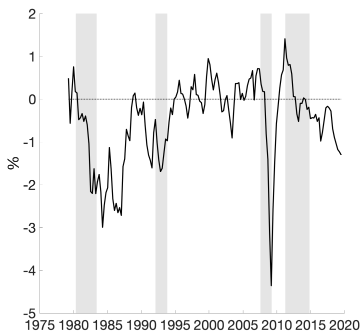

(a) Canada



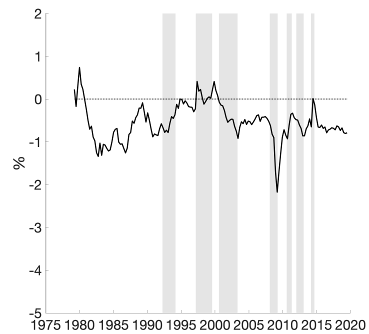

(b) France



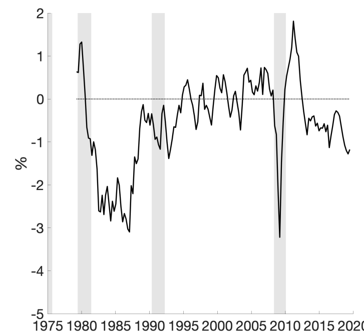

(c) Germany



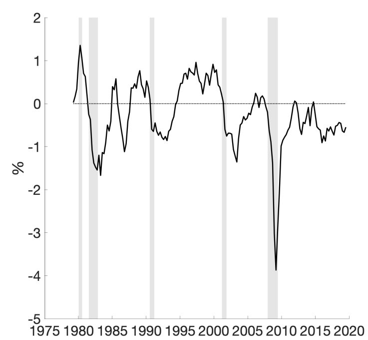

(d) Italy




(e) Japan

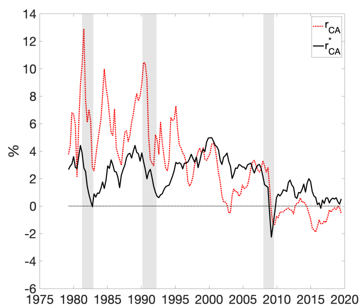



(f) United Kingdom

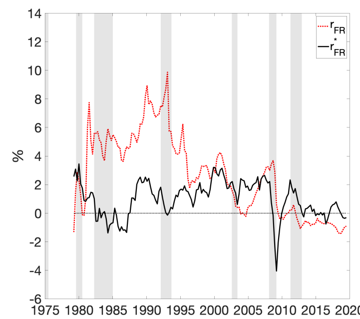



(g) United States

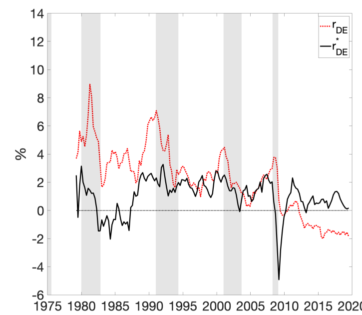
Figure 11: The Real Interest Rate vs The Natural Rate of Interest



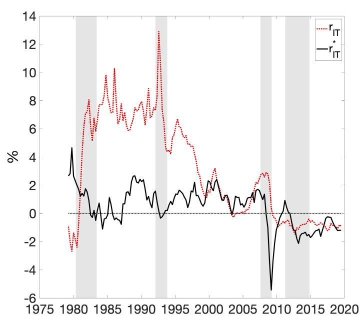
(a) Canada



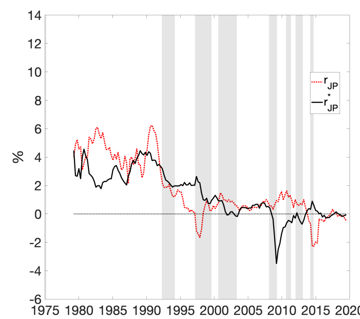
(b) France



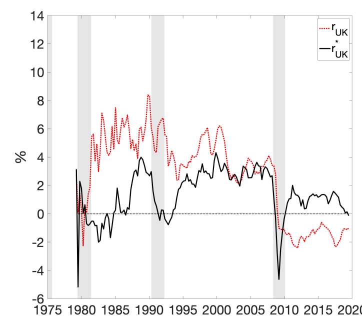
(c) Germany



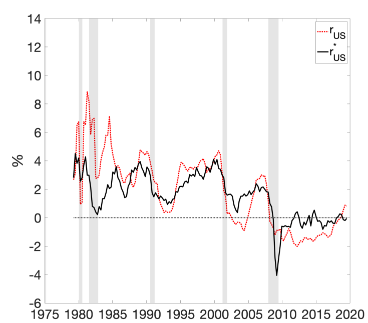
(d) Italy



(e) Japan

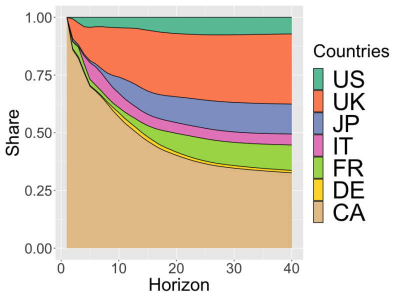


(f) United Kingdom

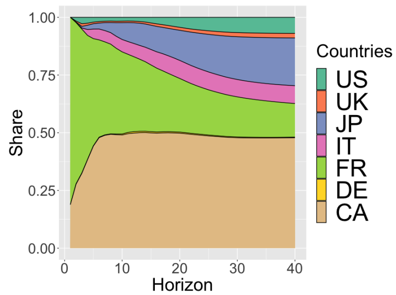


(g) United States

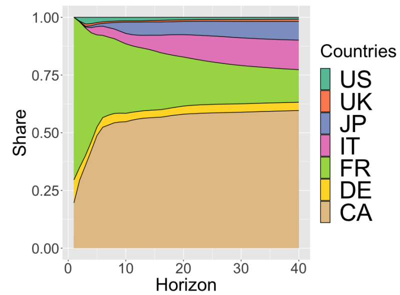
Figure 12: Forecast Error Variance Decompositions



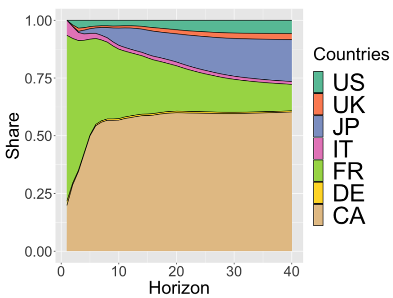

(a) Canada



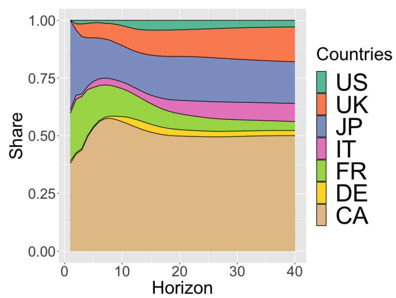

(b) France



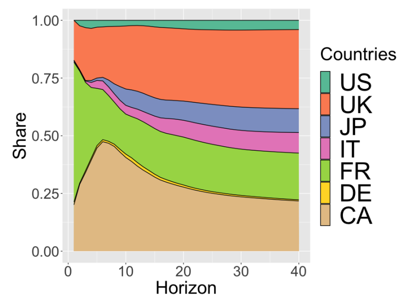

(c) Germany



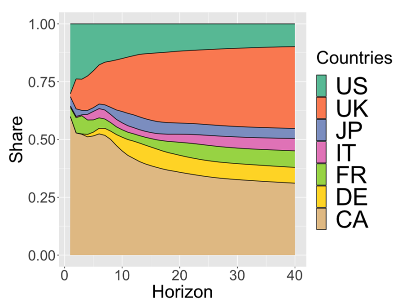

(d) Italy




(e) Japan

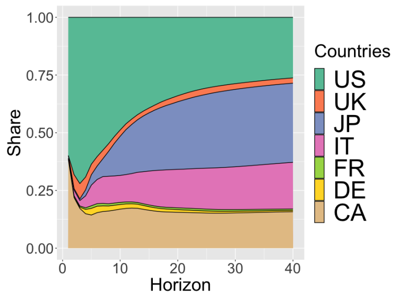



(f) UK

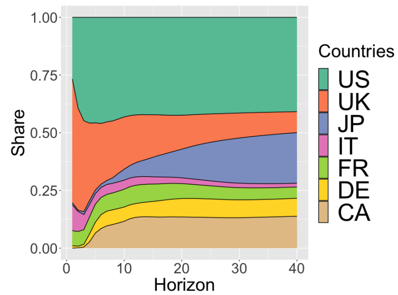



(g) US

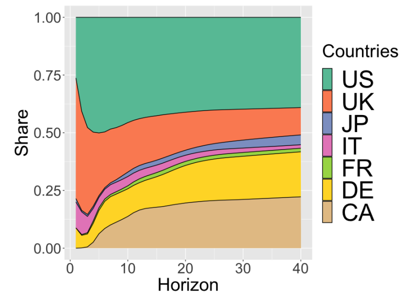
Figure 13: Forecast Error Variance Decomposition (Reverse Order)



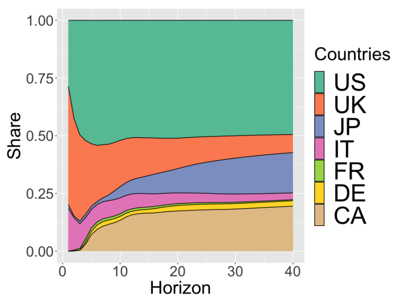

(a) Canada



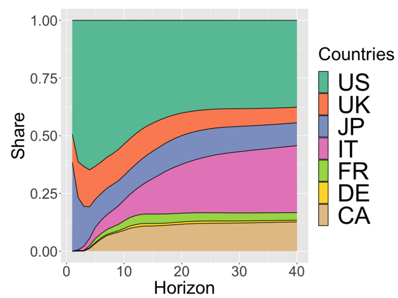

(b) France



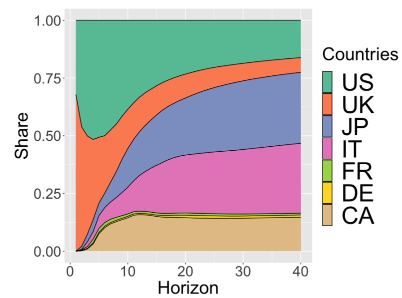

(c) Germany



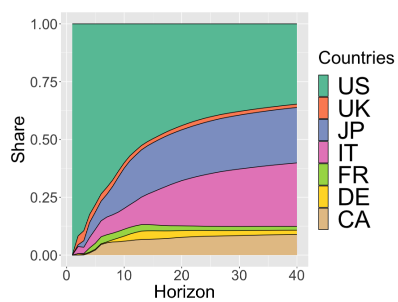

(d) Italy




(e) Japan

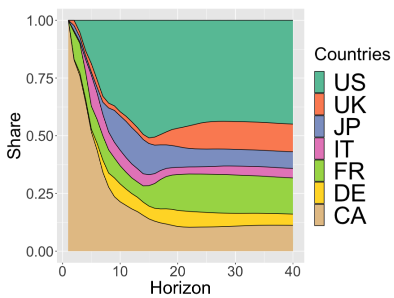



(f) UK

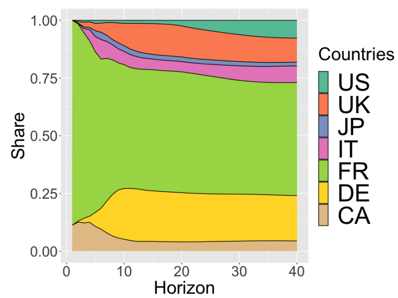



(g) US

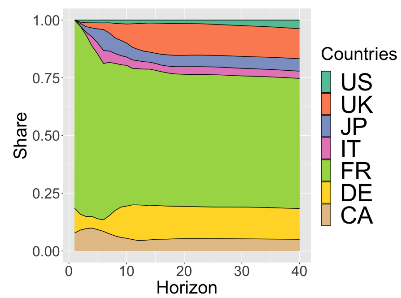
Figure 14: Forecast Error Variance Decomposition (Excluding Financial Crisis)



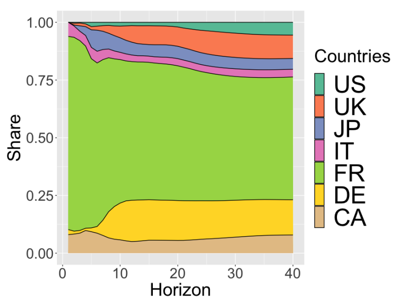

(a) Canada



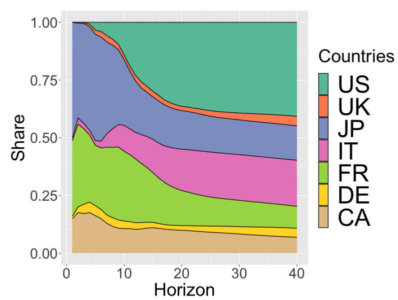

(b) France



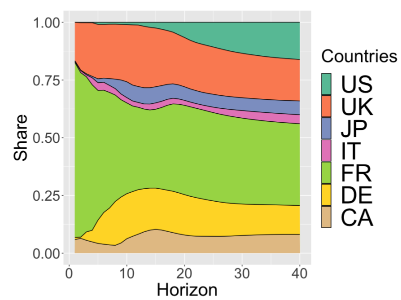

(c) Germany



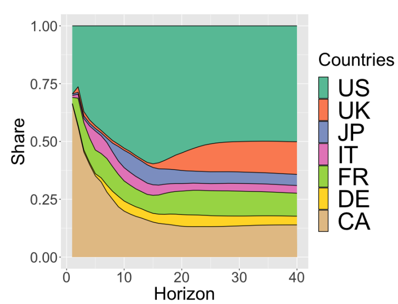

(d) Italy




(e) Japan

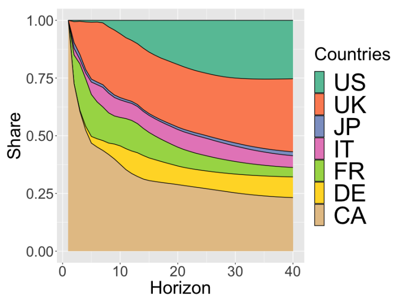



(f) UK

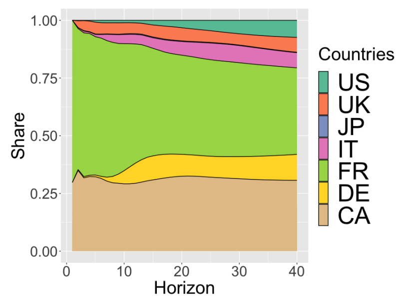



(g) US

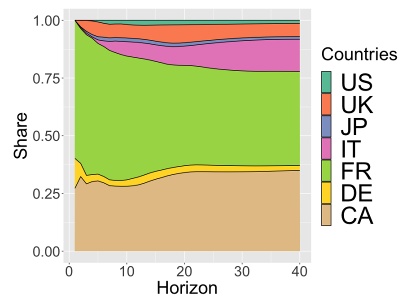
Figure 15: Forecast Error Variance Decomposition (Excluding Energy Crisis)



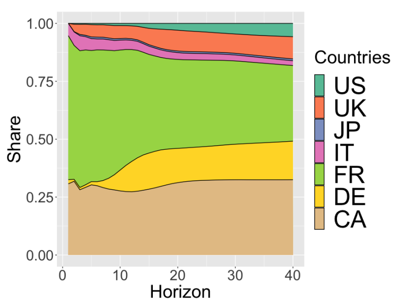

(a) Canada



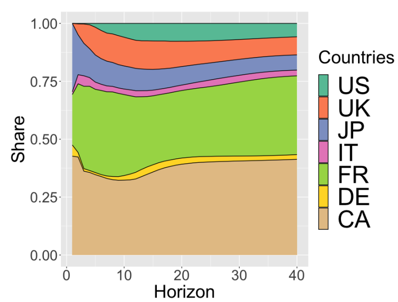

(b) France



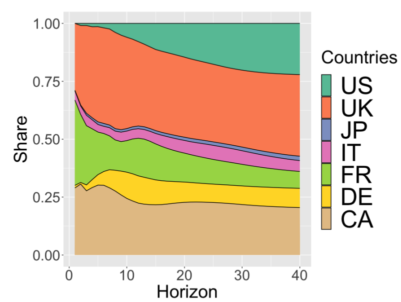

(c) Germany



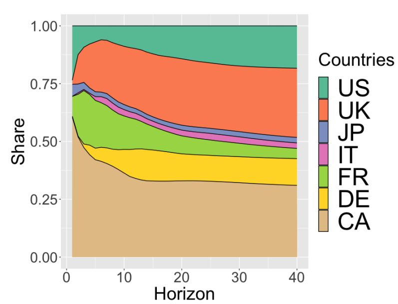

(d) Italy




(e) Japan




(f) UK




(g) US

# List of Tables

Table 1: Correlations of the Natural Rates in the G7 Countries

| | $r^*_{CA}$ | $r^*_{FR}$ | $r^*_{DE}$ | $r^*_{IT}$ | $r^*_{JP}$ | $r^*_{UK}$ | $r^*_{US}$ |
|---|---|---|---|---|---|---|---|
| $r^*_{CA}$ | 1 | | | | | | |
| $r^*_{FR}$ | 0.71 | 1 | | | | | |
| $r^*_{DE}$ | 0.59 | 0.87 | 1 | | | | |
| $r^*_{IT}$ | 0.79 | 0.85 | 0.75 | 1 | | | |
| $r^*_{JP}$ | 0.49 | 0.24 | 0.36 | 0.63 | 1 | | |
| $r^*_{UK}$ | 0.67 | 0.71 | 0.70 | 0.57 | 0.08 | 1 | |
| $r^*_{US}$ | 0.89 | 0.59 | 0.53 | 0.84 | 0.74 | 0.48 | 1 |

The correlations are calculated based on the median estimates of the natural rate of interest.

Table 2: Correlations of Idiosyncratic Component of Natural Rates in the G7 Countries

| | $r^{*idio}_{CA}$ | $r^{*idio}_{FR}$ | $r^{*idio}_{DE}$ | $r^{*idio}_{IT}$ | $r^{*idio}_{JP}$ | $r^{*idio}_{UK}$ | $r^{*idio}_{US}$ |
|---|---|---|---|---|---|---|---|
| $r^{*idio}_{CA}$ | 1 | | | | | | |
| $r^{*idio}_{FR}$ | 0.47 | 1 | | | | | |
| $r^{*idio}_{DE}$ | -0.03 | -0.12 | 1 | | | | |
| $r^{*idio}_{IT}$ | -0.56 | -0.16 | -0.56 | 1 | | | |
| $r^{*idio}_{JP}$ | -0.74 | -0.63 | 0.07 | 0.48 | 1 | | |
| $r^{*idio}_{UK}$ | 0.57 | -0.14 | -0.17 | -0.52 | -0.39 | 1 | |
| $r^{*idio}_{US}$ | -0.12 | -0.01 | -0.28 | 0.42 | 0.07 | -0.01 | 1 |

The correlations are calculated based on the median estimates of the natural rate of interest.

Table 3: Correlations of Natural Rates and Idiosyncratic Components of Natural Rates in Japan, UK, and US

| | $r^*_{JP}$ | $r^*_{UK}$ | $r^*_{US}$ | $r^{*idio}_{JP}$ | $r^{*idio}_{UK}$ | $r^{*idio}_{US}$ |
|---|---|---|---|---|---|---|
| $r^*_{JP}$ | 1 | | | | | |
| $r^*_{UK}$ | 0.63 | 1 | | | | |
| $r^*_{US}$ | 0.80 | 0.84 | 1 | | | |
| $r^{*idio}_{JP}$ | 0.95 | 0.38 | 0.61 | 1 | | |
| $r^{*idio}_{UK}$ | -0.88 | -0.22 | -0.60 | -0.94 | 1 | |
| $r^{*idio}_{US}$ | -0.47 | -0.11 | 0.06 | -0.58 | 0.37 | 1 |

The correlations are calculated based on the median estimates of the natural rate of interest.

Table 4: Correlations of $r^*$s in the G7 Countries Excluding the Financial Crisis

| | $r^*_{CA}$ | $r^*_{FR}$ | $r^*_{DE}$ | $r^*_{IT}$ | $r^*_{JP}$ | $r^*_{UK}$ | $r^*_{US}$ |
|---|---|---|---|---|---|---|---|
| $r^*_{CA}$ | 1 | | | | | | |
| $r^*_{FR}$ | 0.66 | 1 | | | | | |
| $r^*_{DE}$ | 0.55 | 0.78 | 1 | | | | |
| $r^*_{IT}$ | 0.47 | 0.71 | 0.62 | 1 | | | |
| $r^*_{JP}$ | -0.10 | 0.03 | 0.20 | 0.58 | 1 | | |
| $r^*_{UK}$ | 0.64 | 0.50 | 0.62 | 0.23 | -0.19 | 1 | |
| $r^*_{US}$ | 0.82 | 0.57 | 0.48 | 0.70 | 0.32 | 0.32 | 1 |

The correlations are calculated based on the median estimates of the natural rate of interest.

Table 5: Correlations of $r^{*idio}$s in the G7 Countries Excluding the Financial Crisis

| | $r^{*idio}_{CA}$ | $r^{*idio}_{FR}$ | $r^{*idio}_{DE}$ | $r^{*idio}_{IT}$ | $r^{*idio}_{JP}$ | $r^{*idio}_{UK}$ | $r^{*idio}_{US}$ |
|---|---|---|---|---|---|---|---|
| $r^{*idio}_{CA}$ | 1 | | | | | | |
| $r^{*idio}_{FR}$ | 0.31 | 1 | | | | | |
| $r^{*idio}_{DE}$ | 0.03 | -0.07 | 1 | | | | |
| $r^{*idio}_{IT}$ | -0.58 | 0.09 | -0.44 | 1 | | | |
| $r^{*idio}_{JP}$ | -0.80 | -0.46 | -0.09 | 0.54 | 1 | | |
| $r^{*idio}_{UK}$ | 0.66 | -0.27 | -0.10 | -0.73 | -0.54 | 1 | |
| $r^{*idio}_{US}$ | -0.72 | 0.05 | -0.22 | 0.78 | 0.49 | -0.69 | 1 |

The correlations are calculated based on the median estimates of the natural rate of interest.

Table 6: Correlations of $r^*$s in the G7 Countries Excluding the Energy Crisis

| | $r^*_{CA}$ | $r^*_{FR}$ | $r^*_{DE}$ | $r^*_{IT}$ | $r^*_{JP}$ | $r^*_{UK}$ | $r^*_{US}$ |
|---|---|---|---|---|---|---|---|
| $r^*_{CA}$ | 1 | | | | | | |
| $r^*_{FR}$ | 0.78 | 1 | | | | | |
| $r^*_{DE}$ | 0.59 | 0.85 | 1 | | | | |
| $r^*_{IT}$ | 0.80 | 0.91 | 0.86 | 1 | | | |
| $r^*_{JP}$ | 0.61 | 0.70 | 0.82 | 0.87 | 1 | | |
| $r^*_{UK}$ | 0.85 | 0.81 | 0.66 | 0.77 | 0.55 | 1 | |
| $r^*_{US}$ | 0.91 | 0.83 | 0.74 | 0.93 | 0.80 | 0.82 | 1 |

The correlations are calculated based on the median estimates of the natural rate of interest.

Table 7: Correlations of $r^{*idio}$s in the G7 Countries Excluding the Energy Crisis

| | $r^{*idio}_{CA}$ | $r^{*idio}_{FR}$ | $r^{*idio}_{DE}$ | $r^{*idio}_{IT}$ | $r^{*idio}_{JP}$ | $r^{*idio}_{UK}$ | $r^{*idio}_{US}$ |
|---|---|---|---|---|---|---|---|
| $r^{*idio}_{CA}$ | 1 | | | | | | |
| $r^{*idio}_{FR}$ | 0.49 | 1 | | | | | |
| $r^{*idio}_{DE}$ | -0.40 | -0.14 | 1 | | | | |
| $r^{*idio}_{IT}$ | -0.34 | -0.50 | -0.53 | 1 | | | |
| $r^{*idio}_{JP}$ | -0.69 | -0.63 | 0.19 | 0.53 | 1 | | |
| $r^{*idio}_{UK}$ | 0.60 | 0.13 | -0.39 | -0.20 | -0.45 | 1 | |
| $r^{*idio}_{US}$ | 0.09 | -0.15 | -0.56 | 0.44 | -0.02 | 0.34 | 1 |

The correlations are calculated based on the median estimates of the natural rate of interest.

Table 8: Posterior Estimates: Canada

| Parameter | Density | Parameter1 | Parameter2 | Posterior Median |
|---|---|---|---|---|
| $a_{y1,CA}$ | Normal | MLE | 0.3 | 1.46 |
| | | | | (1.43, 1.48) |
| $a_{y2,CA}$ | Normal | MLE | 0.3 | -0.54 |
| | | | | (-0.56, -0.53) |
| $a_{r,CA}$ | Normal | min(MLE,-0.01) | 0.3 | -0.08 |
| | | | | (-0.10, -0.07) |
| $b_{\pi,CA}$ | Beta | MLE | 0.3 | 0.45 |
| | | | | (0.41, 0.47) |
| $b_{y,CA}$ | Normal | min(MLE,0.5) | 0.3 | 0.16 |
| | | | | (0.11, 0.20) : |
| $\sigma_{y_{CA}}$ | Inverse-Gamma | MLE | 0.1 | 0.47 |
| | | | | (0.43, 0.49) |
| $\sigma_{\pi_{CA}}$ | Inverse-Gamma | MLE | 0.1 | 1.47 |
| | | | | (1.42, 1.49) |
| $\sigma^*_{CA}$ | Inverse-Gamma | MLE | 0.1 | 0.33 |
| | | | | (0.32, 0.35) |
| $\sigma_{g_{CA}}$ | Inverse-Gamma | 0.03 | 0.03 | 0.06 |
| | | | | (0.04, 0.06) |
| $\sigma_{z_{CA}}$ | Inverse-Gamma | 0.1 | 0.1 | 0.20 |
| | | | | (0.18, 0.21) |

Parameter 1 is the mean of the normal distribution, the mean of the beta distribution, and the mode value of the inverse-gamma distribution. Parameter 2 is the standard deviation of the all of the distributions. MLE represents the estimates from the maximum likelihood on a country-by-country basis using the three-step estimation method in Holston et al. 2017. In the parenthesis, the number on the left represents the 25th posterior draw and the number on the right represent the 75th posterior draw.

Table 9: Posterior Estimates: France

| Parameter | Density | Parameter1 | Parameter2 | Posterior Median |
|---|---|---|---|---|
| $a_{y1,FR}$ | Normal | MLE | 0.3 | 1.28 |
| | | | | (1.26, 1.31) |
| $a_{y2,FR}$ | Normal | MLE | 0.3 | -0.44 |
| | | | | (-0.47, -0.41) |
| $a_{r,FR}$ | Normal | min(MLE,-0.01) | 0.3 | -0.05 |
| | | | | (-0.06, -0.04) |
| $b_{\pi,FR}$ | Beta | MLE | 0.3 | 0.63 |
| | | | | (0.58, 0.67) |
| $b_{y,FR}$ | Normal | min(MLE,0.5) | 0.3 | 0.14 |
| | | | | (0.13, 0.16) |
| $\sigma_{y_{FR}}$ | Inverse-Gamma | MLE | 0.1 | 0.18 |
| | | | | (0.16, 0.19) |
| $\sigma_{\pi_{FR}}$ | Inverse-Gamma | MLE | 0.1 | 0.97 |
| | | | | (0.96, 0.98) |
| $\sigma_{FR}^{*}$ | Inverse-Gamma | MLE | 0.1 | 0.26 |
| | | | | (0.25, 0.27) |
| $\sigma_{g_{FR}}$ | Inverse-Gamma | 0.03 | 0.03 | 0.03 |
| | | | | (0.02, 0.04) |
| $\sigma_{z_{FR}}$ | Inverse-Gamma | 0.1 | 0.1 | 0.20 |
| | | | | (0.18, 0.24) |

Parameter 1 is the mean of the normal distribution, the mean of the beta distribution, and the mode value of the inverse-gamma distribution. Parameter 2 is the standard deviation of the all of the distributions. MLE represents the estimates from the maximum likelihood on a country-by-country basis using the three-step estimation method in Holston et al. 2017. In the parenthesis, the number of the left represents the 25th posterior draw and the number of the right represent the 75th posterior draw.

Table 10: Posterior Estimates: Germany

| Parameter | Density | Parameter1 | Parameter2 | Posterior Median |
|---|---|---|---|---|
| $a_{y1,DE}$ | Normal | MLE | 0.3 | 1.60 |
| | | | | ( 1.54 , 1.63 ) |
| $a_{y2,DE}$ | Normal | MLE | 0.3 | -0.74 |
| | | | | ( -0.79 , -0.70 ) |
| $a_{r,DE}$ | Normal | min(MLE,-0.01) | 0.3 | -0.01 |
| | | | | ( -0.02 , -0.01 ) |
| $b_{\pi,DE}$ | Beta | MLE | 0.3 | 0.44 |
| | | | | ( 0.38 , 0.48 ) |
| $b_{y,DE}$ | Normal | min(MLE,0.5) | 0.3 | 0.33 |
| | | | | ( 0.29 , 0.40 ) |
| $\sigma_{y_{DE}}$ | Inverse-Gamma | MLE | 0.1 | 0.18 |
| | | | | ( 0.17 , 0.20 ) |
| $\sigma_{\pi_{DE}}$ | Inverse-Gamma | MLE | 0.1 | 1.23 |
| | | | | ( 1.21 , 1.29 ) |
| $\sigma^{*}_{DE}$ | Inverse-Gamma | MLE | 0.1 | 0.82 |
| | | | | ( 0.80 , 0.83 ) |
| $\sigma_{g_{DE}}$ | Inverse-Gamma | 0.03 | 0.03 | 0.03 |
| | | | | ( 0.02 , 0.04 ) |
| $\sigma_{z_{DE}}$ | Inverse-Gamma | 0.1 | 0.1 | 0.30 |
| | | | | ( 0.29 , 0.30 ) |

Parameter 1 is the mean of the normal distribution, the mean of the beta distribution, and the mode value of the inverse-gamma distribution. Parameter 2 is the standard deviation of the all of the distributions. MLE represents the estimates from the maximum likelihood on a country-by-country basis using the three-step estimation method in Holston et al. 2017. In the parenthesis, the number of the left represents the 25th posterior draw and the number of the right represent the 75th posterior draw.

Table 11: Posterior Estimates: Italy

| Parameter | Density | Parameter1 | Parameter2 | Posterior Median |
|---|---|---|---|---|
| $a_{y1,IT}$ | Normal | MLE | 0.3 | 1.58 |
| | | | | ( 1.54 , 1.61 ) |
| $a_{y2,IT}$ | Normal | MLE | 0.3 | -0.74 |
| | | | | ( -0.77 , -0.72 ) |
| $a_{r,IT}$ | Normal | min(MLE,-0.01) | 0.3 | -0.06 |
| | | | | ( -0.07 , -0.04 ) |
| $b_{\pi,IT}$ | Beta | MLE | 0.3 | 0.59 |
| | | | | ( 0.56 , 0.61 ) |
| $b_{y,IT}$ | Normal | min(MLE,0.5) | 0.3 | 0.14 |
| | | | | ( 0.08 , 0.17 ) |
| $\sigma_{y_{IT}}$ | Inverse-Gamma | MLE | 0.1 | 0.16 |
| | | | | ( 0.16 , 0.18 ) |
| $\sigma_{\pi_{IT}}$ | Inverse-Gamma | MLE | 0.1 | 1.36 |
| | | | | ( 1.34 , 1.37 ) |
| $\sigma^*_{IT}$ | Inverse-Gamma | MLE | 0.1 | 0.50 |
| | | | | ( 0.47 , 0.51 ) |
| $\sigma_{g_{IT}}$ | Inverse-Gamma | 0.03 | 0.03 | 0.04 |
| | | | | ( 0.03 , 0.04 ) |
| $\sigma_{z_{IT}}$ | Inverse-Gamma | 0.1 | 0.1 | 0.07 |
| | | | | ( 0.06 , 0.09 ) |

Parameter 1 is the mean of the normal distribution, the mean of the beta distribution, and the mode value of the inverse-gamma distribution. Parameter 2 is the standard deviation of the all of the distributions. MLE represents the estimates from the maximum likelihood on a country-by-country basis using the three-step estimation method in Holston et al. 2017. In the parenthesis, the number of the left represents the 25th posterior draw and the number of the right represent the 75th posterior draw.

Table 12: Posterior Estimates: Japan

| Parameter | Density | Parameter1 | Parameter2 | Posterior Median |
|---|---|---|---|---|
| $a_{y1,JP}$ | Normal | MLE | 0.3 | 1.44 |
| | | | | ( 1.33 , 1.48 ) |
| $a_{y2,JP}$ | Normal | MLE | 0.3 | -0.79 |
| | | | | ( -0.84 , -0.69 ) |
| $a_{r,JP}$ | Normal | min(MLE,-0.01) | 0.3 | -0.01 |
| | | | | ( -0.02 , -0.01 ) |
| $b_{\pi,JP}$ | Beta | MLE | 0.3 | 0.38 |
| | | | | ( 0.36 , 0.40 ) |
| $b_{y,JP}$ | Normal | min(MLE,0.5) | 0.3 | 0.62 |
| | | | | ( 0.56 , 0.66 ) |
| $\sigma_{y_{JP}}$ | Inverse-Gamma | MLE | 0.1 | 0.03 |
| | | | | ( 0.02 , 0.04 ) |
| $\sigma_{\pi_{JP}}$ | Inverse-Gamma | MLE | 0.1 | 1.18 |
| | | | | ( 1.16 , 1.23 ) |
| $\sigma^*_{JP}$ | Inverse-Gamma | MLE | 0.1 | 0.93 |
| | | | | ( 0.91 , 0.94 ) |
| $\sigma_{g_{JP}}$ | Inverse-Gamma | 0.03 | 0.03 | 0.02 |
| | | | | ( 0.02 , 0.03 ) |
| $\sigma_{z_{JP}}$ | Inverse-Gamma | 0.1 | 0.1 | 0.23 |
| | | | | ( 0.21 , 0.24 ) |

Parameter 1 is the mean of the normal distribution, the mean of the beta distribution, and the mode value of the inverse-gamma distribution. Parameter 2 is the standard deviation of the all of the distributions. MLE represents the estimates from the maximum likelihood on a country-by-country basis using the three-step estimation method in Holston et al. 2017. In the parenthesis, the number of the left represents the 25th posterior draw and the number of the right represent the 75th posterior draw.

Table 13: Posterior Estimates: United Kingdom

| Parameter | Density | Parameter1 | Parameter2 | Posterior Median |
|---|---|---|---|---|
| $a_{y1,UK}$ | Normal | MLE | 0.3 | 1.51 |
| | | | | ( 1.49 , 1.55 ) |
| $a_{y2,UK}$ | Normal | MLE | 0.3 | -0.68 |
| | | | | ( -0.71 , -0.66 ) |
| $a_{r,UK}$ | Normal | min(MLE,-0.01) | 0.3 | -0.03 |
| | | | | ( -0.04 , -0.02 ) |
| $b_{\pi,UK}$ | Beta | MLE | 0.3 | 0.37 |
| | | | | ( 0.35 , 0.40 ) |
| $b_{y,UK}$ | Normal | min(MLE,0.5) | 0.3 | 0.76 |
| | | | | ( 0.71 , 0.82 ) |
| $\sigma_{y_{UK}}$ | Inverse-Gamma | MLE | 0.1 | 0.25 |
| | | | | ( 0.23 , 0.26 ) |
| $\sigma_{\pi_{UK}}$ | Inverse-Gamma | MLE | 0.1 | 2.22 |
| | | | | ( 2.21 , 2.23 ) |
| $\sigma^*_{UK}$ | Inverse-Gamma | MLE | 0.1 | 0.47 |
| | | | | ( 0.46 , 0.48 ) |
| $\sigma_{g_{UK}}$ | Inverse-Gamma | 0.03 | 0.03 | 0.06 |
| | | | | ( 0.06 , 0.07 ) |
| $\sigma_{z_{UK}}$ | Inverse-Gamma | 0.1 | 0.1 | 0.09 |
| | | | | ( 0.08 , 0.10 ) |

PParameter 1 is the mean of the normal distribution, the mean of the beta distribution, and the mode value of the inverse-gamma distribution. Parameter 2 is the standard deviation of the all of the distributions. MLE represents the estimates from the maximum likelihood on a country-by-country basis using the three-step estimation method in Holston et al. 2017. In the parenthesis, the number of the left represents the 25th posterior draw and the number of the right represent the 75th posterior draw.

Table 14: Posterior Estimates: United States

| Parameter | Density | Parameter1 | Parameter2 | Posterior Median |
|---|---|---|---|---|
| $a_{y1,US}$ | Normal | MLE | 0.3 | 1.42 |
| | | | | ( 1.41 , 1.44 ) |
| $a_{y2,US}$ | Normal | MLE | 0.3 | -0.54 |
| | | | | ( -0.56 , -0.53 ) |
| $a_{r,US}$ | Normal | min(MLE,-0.01) | 0.3 | -0.15 |
| | | | | ( -0.16 , -0.12 ) |
| $b_{\pi,US}$ | Beta | MLE | 0.3 | 0.26 |
| | | | | ( 0.22 , 0.29 ) |
| $b_{y,US}$ | Normal | min(MLE,0.5) | 0.3 | 0.15 |
| | | | | ( 0.12 , 0.16 ) |
| $\sigma_{y_{US}}$ | Inverse-Gamma | MLE | 0.1 | 0.21 |
| | | | | ( 0.20 , 0.22 ) |
| $\sigma_{\pi_{US}}$ | Inverse-Gamma | MLE | 0.1 | 0.72 |
| | | | | ( 0.71 , 0.74 ) |
| $\sigma^*_{US}$ | Inverse-Gamma | MLE | 0.1 | 0.49 |
| | | | | ( 0.48 , 0.50 ) |
| $\sigma_{g_{US}}$ | Inverse-Gamma | 0.03 | 0.03 | 0.06 |
| | | | | ( 0.06 , 0.06 ) |
| $\sigma_{z_{US}}$ | Inverse-Gamma | 0.1 | 0.1 | 0.09 |
| | | | | ( 0.08 , 0.09 ) |

Parameter 1 is the mean of the normal distribution, the mean of the beta distribution, and the mode value of the inverse-gamma distribution. Parameter 2 is the standard deviation of the all of the distributions. MLE represents the estimates from the maximum likelihood on a country-by-country basis using the three-step estimation method in Holston et al. 2017. In the parenthesis, the number of the left represents the 25th posterior draw and the number of the right represent the 75th posterior draw.

Table 15: Posterior Estimates: Common Parameters

| Parameter | Density | Parameter1 | Parameter2 | Posterior Median |
|---|---|---|---|---|
| $\sigma_{g^{common}}$ | Inverse-Gamma | 0.03 | 0.03 | 0.05 |
| | | | | ( 0.05 , 0.05 ) |
| $\sigma_{z^{common}}$ | Inverse-Gamma | 0.1 | 0.1 | 0.12 |
| | | | | ( 0.10 , 0.17 ) |
| $\sigma_{g^{north-america}}$ | Inverse-Gamma | 0.03 | 0.03 | 0.06 |
| | | | | ( 0.06 , 0.07 ) |
| $\sigma_{z^{north-america}}$ | Inverse-Gamma | 0.1 | 0.1 | 0.17 |
| | | | | ( 0.16 , 0.18 ) |
| $\sigma_{g^{europe}}$ | Inverse-Gamma | 0.03 | 0.03 | 0.08 |
| | | | | ( 0.07 , 0.09 ) |
| $\sigma_{z^{europe}}$ | Inverse-Gamma | 0.1 | 0.1 | 0.26 |
| | | | | ( 0.25 , 0.27 ) |

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Parameter 1 is the mean of the normal distribution, the mean of the beta distribution, and the mode value of the inverse-gamma distribution. Parameter 2 is the standard deviation of the all of the distributions. MLE represents the estimates from the maximum likelihood on a country-by-country basis using the three-step estimation method in Holston et al. 2017. In the parenthesis, the number of the left represents the 25th posterior draw and the number of the right represent the 75th posterior draw.

Table 16: Prior Specification for the Country-by-Country Estimation

| Parameter | Domain | Density | Parameter1 | Parameter2 |
|---|---|---|---|---|
| $a_{y1,i}$ | $[0, \infty)$ | Normal | MLE | 0.3 |
| $a_{y2,i}$ | R | Normal | MLE | 0.3 |
| $a_{r,i}$ | $(-\infty, -0.0025)$ | Normal | min(MLE,-0.01) | 0.3 |
| $b_{\pi,i}$ | [0,1] | Beta | MLE | 0.3 |
| $b_{y,i}$ | $(0.025, \infty)$ | Normal | min(MLE,0.5) | 0.3 |
| $\sigma_{y_i}$ | $(0,\infty)$ | Inverse-Gamma | MLE | 0.1 |
| $\sigma_{\pi_i}$ | $(0,\infty)$ | Inverse-Gamma | MLE | 0.1 |
| $\sigma_{y_i^*}$ | $(0,\infty)$ | Inverse-Gamma | MLE | 0.1 |
| $\sigma_{g_i}$ | $(0,\sigma_{z_i})$ | Inverse-Gamma | 0.03 | 0.03 |
| $\sigma_{z_i}$ | $(0,\infty)$ | Inverse-Gamma | 0.1 | 0.1 |

$i = \{CA, FR, DE, IT, JP, UK, US\}$. Parameter 1 is the mean of the normal distribution, the mean of the beta distribution, and the mode value of the inverse-gamma distribution. Parameter 2 is the standard deviation of the all of the distributions.

Table 17: Prior Specification for the Stationary Processes on the Preference Shifters

| Parameter | Domain | Density | Parameter1 | Parameter2 |
|---|---|---|---|---|
| $\sigma_{z_i}$ | (0,∞) | Inverse-Gamma | 0.5 | 0.3 |
| $\sigma_{z^{common}}$ | (0, ∞) | Inverse-Gamma | 0.5 | 0.3 |
| $\sigma_{z^{north-america}}$ | (0,∞) | Inverse-Gamma | 0.5 | 0.3 |
| $\sigma_{z^{europe}}$ | (0,∞) | Inverse-Gamma | 0.5 | 0.3 |
| $\rho_i$ | [0,1] | Beta | 0.5 | 0.2 |
| $\rho^{common}$ | [0,1] | Beta | 0.5 | 0.2 |
| $\rho^{north-america}$ | [0,1] | Beta | 0.5 | 0.2 |
| $\rho^{europe}$ | [0,1] | Beta | 0.5 | 0.2 |

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$i = \{CA, FR, DE, IT, JP, UK, US\}$. $\rho$ is the AR(1) coefficient. Parameter 1 is the mean of the normal distribution, the mean of the beta distribution, and the mode value of the inverse-gamma distribution. Parameter 2 is the standard deviation of the all of the distributions.