International Comovement of r^*: A Case Study of the G7 Countries

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Abstract

The natural rate of interest, r^*, is an important input to determine the appropriate monetary policy stance. Commonly, the measurement is estimated on a single country basis, which ignores the international factors that may affect r^*. However, expanding to a multiple country model adds substantive model complexity. In this paper, I exploit a Bayesian method to build a multi-country state space model, which is an extension of Holston et al. (2017), to jointly estimate r^* for the G7 countries. Furthermore, in the process of estimating the model, I decompose the country level r^* into common, regional, and idiosyncratic components and identity the dynamics of each component. I find that across the G7 countries r^* has been declining since the 1990s and is driven by the common component. I also find the contribution of the idiosyncratic components to r^* are minor. These results suggest a synchronization of the natural rate of interest across countries since the 1990s, consistent with the findings in Del Negro et al., 2019 where the low natural rate is due to a rise in the demand for the safe and liquid assets.

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1 Introduction

The natural rate of interest, also known as the equilibrium real interest rate or r^*, is the real short-term interest rate at which real GDP equals its potential GDP without transitory shocks (Williams, 2003). Although it is not directly observed, the value of r^* is an important input into monetary policy decisions, as it provides a benchmark for the stance of monetary policy (Taylor, 1993). The natural rate of interest (henceforth the natural rate) is commonly estimated on a country-by-country basis and thus the international dimension[1] has often been ignored. Despite that expanding a single country model to a multi-county model adds substantive model complexity, incorporating a global component would allow policy makers to determine to what extent r^* is influenced by local and global factors, and whether or not the integration of r^* has accelerated after the financial crisis.

In this paper, I build a multi-country state space model, which is an extention of Holston et al. (2017), to jointly estimate the natural rate for the G7 countries, namely Canada, France, Germany, Italy, Japan, the UK, and the US, exploiting a Bayesian method. In the process of estimating the model, I also decompose the country level r^* into common, regional, and idiosyncratic components to identify the dynamics of each component and investigate contributions of each component to r^* for each country.

This paper offers several contributions to the literature. First, the joint estimation of the natural rate takes into account the international components. Previous studies focus on estimating the natural rate on an individual country basis (e.g. Holston et al., 2017, Hamilton et al., 2016, Kiley, 2015 and many others) and joint consideration is scarce. Due to the high integration of macroeconomic variables across countries, estimating the variable by only considering the domestic factors may lead to misspecification and a biased estimation. This paper sheds light on joint estimation in the literature of measuring r^*. Furthermore, by exploiting the Holston et al. (2017) framework, I also jointly estimate the potential output.

Second, this paper provides the decomposition of the natural rate into common, regional, and idiosyncratic components. In the literature on international business cycles, comovement of macroeconomic variables across countries has been well investigated (e.g. Gregory et al., 1997 and Kose

[1] It may be not only essential in a large open economy but also in a small open economy as foreign factors dramatically influence those economies (e.g. Grossman et al., 2019). Also international finance plays a role in influencing r^*. For example, a rise in popularity in safe assets as well as the spillover effect influence r^* through the international finance market (e.g. Glick, 2019).

et al., 2003). Recently, the topic extends to the real interest rate (e.g. Del Negro et al., 2019). In this context, this paper adds the decomposition of r^* of the G7 countries to the literature. In addition, this paper also investigates whether only a subset of the G7 countries possess a specific dynamic (e.g. Mitra and Sinclair, 2012).

Third, this paper examines the roles of domestic and international components of the natural rate in the context of monetary policy. The importance of the natural rate for monetary policy has been widely explored (e.g. Barsky et al., 2014 and Canzoneri et al., 2015). An accurate measure of the natural rate is crucial as the optimal policy varies with the estimated natural rate (e.g. Brand et al., 2018 and Amato, 2005). This paper includes international factors when estimating the natural rates and provides the relative importance of the components of the natural rates, which provides contributions to an efficient monetary policy operation.

I use quarterly frequency[2] data for output, the price level, and the short term nominal interest rate for the G7 countries. The samples range from 1979Q4 to 2019Q2. I set up a multi-country state space model which is an extension of Holston et al. (2017) and estimate the parameters using a random walk Metropolis-Hastings algorithm. I then estimate the natural rate using the Kalman filter. At the same time, I decompose the natural rate into common, regional, and idiosyncratic components.

I find that the natural rate has generally tended to decline for the G7 countries, especially after the 1990s. For example, in the US, the estimated r^* is 3.54% in 1979 and -0.02% in 2019.[3] This is consistent with the overwhelming evidence in the literature (e.g. Holston et al., 2017; Kiley, 2015; Juselius et al., 2016; and many others). I also find that the natural rate behaves in a pro-cyclical manner,[4] with large drops during both the energy crisis in the 1980s and the financial crisis in 2007. Furthermore, I find the estimates of the natural rate to be lower than the individual estimates that are found in the literature (e.g. Holston et al., 2017 and Kiley, 2015). In a joint country estimation, Kiley (2019) uncovers a similar finding, indicating that missing international factors may lead to

[2]The quarterly frequency is a long period for measuring interest rate behavior. However, the natural rate is a theoretical "long run" short-term interest rate consistent with potential GDP and a stable inflation rate. Because r^* is a long run variable, within a quarter, dramatic changes in r^* should be sufficiently captured. Also, quarterly measures of r^* match the frequency of GDP. Understanding the dynamics of r^* along with potential GDP provides a benchmark stance of monetary policy, which complements the high frequency of monetary policy.

[3]Negative r^* can happen if the interest rate at which loan supply (savings) equals loan demand (investment) is negative. This can be possible if an economy is experiencing a population decline (and the resulting decline in investment) or a rise in life expectancy (a rise in savings). A number of studies have found a negative r^* (e.g. Brand et al. (2018) in the Euro area and Okazaki and Sudo (2018) in Japan).

[4]This behavior is also seen in Holston et al. (2017).

the overestimation in the previous individual country estimates.

The decomposition exercise reveals that the downward tendency of r^* is mainly driven by the common component of the natural rate. This does not neglect the role of the regional component: some large drops of the natural rates in the countries in Europe are driven by the regional component. With regard to the idiosyncratic component, I find that the contributions to the country-level natural rate are minimal. This suggests a synchronization of r^* across the G7 countries.

In the robustness analysis, I estimate a three-country state space model (Japan, the UK, and the US) with a longer sample size. Here, I also find that the common components drive the dynamics of r^* and idiosyncratic components do not play a major role. Also, when estimating the model for the G7 countries with sub-sample periods, I find that the synchronization of r^* accelerated after the financial crisis, consistent with the finding in Del Negro et al. (2019).

The rest of this paper is organized as follows: Section 2 describes the datasets that are used, Section 3 outlines the methodology (including the state space model and Bayesian estimation), Section 4 presents the results, Section 5 checks robustness, and finally Section 6 concludes.

2 Data

This study analyzes the G7 countries, notably Canada, France, Germany, Italy, Japan, the UK, and the US. Data are of quarterly frequency and cover 1979Q4-2019Q2 based on data availability. I use the following three variables for each country: real GDP, the core Consumer Price Index (CPI) [5], and the 3-month nominal interest rate.[6] Real GDP and the core CPI are seasonally adjusted. The data are obtained from the Federal Reserve Economic Data, OECD Data, Datastream, and central banks from these countries.

[5] The core price index of the Personal Consumption Expenditures (PCE) for the US

[6] I use the 3-month maturity interbank rates for Canada, France, Germany, and Italy and use the 3-month maturity government bond yields for the UK and US. I use the 3-month maturity money market rate from 1989Q1 in Japan. Prior to this period in Japan, I use the 10 year maturity government bond yield due to data availability.

3 Methodology

3.1 The Empirical Model

I set up the following state space model, which is an extension of Holston et al. (2017). The model consists of the following measurement equations:

$$\begin{cases} \tilde{y}_{i,t} = a_{y1,i}\tilde{y}_{i,t-1} + a_{y2,i}\tilde{y}_{i,t-2} + \frac{a_{r,i}}{2}(\tilde{r}_{i,t-1} + \tilde{r}_{i,t-2}) + \epsilon_{t,\tilde{y}_i} \\ \pi_{i,t} = b_{\pi,i}\pi_{i,t-1} + (1 - b_{\pi,i})\pi_{i,t-2,4} + b_{y,i}\tilde{y}_{i,t-1} + \epsilon_{t,\pi_i} \end{cases} \tag{1}$$

where $i = \{CA, FR, DE, IT, JP, UK, US\}$, the first equation is the reduced form IS equation and the second equation is the reduced form Phillips curve equation. $\tilde{y}_{i,t} = y_{i,t} - y^*_{i,t}$ represents output gap (log of real GDP minus log of potential GDP), $\tilde{r}_{i,t} = r_{i,t} - r^*_{i,t}$ represents a deviation of real interest rate from the natural rate, $\pi_{i,t}$ represents the inflation rate, and $\pi_{i,t-2,4} = \frac{1}{3}\sum_{j=t-4}^{t-2}\pi_{i,j}$ represents the average of the second to fourth lags of the inflation rates. $\epsilon_{t,\tilde{y}_i}$ and ϵ_{t,π_i} captures the transitory shocks to the measurement equations. Here, the real interest rate is the ex-ante real interest rate: $r_{i,t} = \text{nominal interest rate}_{i,t} - \mathbb{E}_{t-1}[\pi_t]$.[7]

I assume that the natural rate for each country has the following law of motion:

$$\begin{aligned} r^*_{i,t} = &\quad r_t^{*common} + r_t^{*region} + r_{i,t}^{*idiosyncratic} \\ = &\quad g_t^{*common} + z_t^{*common} + g_t^{*region} + z_t^{*region} + g_{i,t}^{*idiosyncratic} + z_{i,t}^{*idiosyncratic} \end{aligned} \tag{2}$$

where $i = \{CA, FR, DE, IT, JP, UK, US\}$, the first equation specifies that the natural rate is a sum of common, regional, and idiosyncratic components. The common component captures the dynamics of r^* for all of the G7 countries. The regional component captures the dynamics of a specific group[8]: North America (Canada and the US) and Europe (France, Germany, Italy, and the UK). Note that I cannot identify the regional component or the idiosyncratic component for Japan given that it is the only Asian country in this study. Thus, the idiosyncratic component of the natural rate in Japan includes both regional and idiosyncratic components.

To obtain the second equation, I assume that each common, regional, and idiosyncratic compo-

[7] where $\mathbb{E}_{t-1}[\pi_t]$ is a four-quarter moving average of past inflation rates as a proxy for inflation expectations based on Holston et al. (2017).

[8] As in the Gravity equation, the closer the countries, the higher the trade flow. Thus proximity is one measure of economic integration. Specifying regional components are found in previous studies (e.g. Kose et al., 2003 and Mansour, 2003).

nent of the natural rate is a sum of the respective output growth, g, and preference shifter, z. This specification relies on Holston et al. (2017) based on the theoretical link between the natural rate and output growth.[9]

Next, the model consists of the following state equations:

$$\begin{cases} y^*_{i,t} & = & y^*_{i,t-1} + g^{common}_{t-1} + g^{region}_{t-1} + g^{idiosyncratic}_{i,t-1} & +\epsilon_{t,y^*_i} \\ g^{common}_t & = & g^{common}_{t-1} & +\epsilon_{t,g^{common}} \\ g^{region}_t & = & g^{region}_{t-1} & +\epsilon_{t,g^{region}} \\ g^{idiosyncratic}_{i,t} & = & z^{idiosyncratic}_{i,t-1} & +\epsilon_{t,g^{idiosyncratic}_i} \\ z^{common}_t & = & z^{common}_{t-1} & +\epsilon_{t,z^{common}} \\ z^{region}_t & = & z^{region}_{t-1} & +\epsilon_{t,z^{region}} \\ z^{idiosyncratic}_{i,t} & = & z^{idiosyncratic}_{i,t-1} & +\epsilon_{t,z^{idiosyncratic}_i} \end{cases} \tag{3}$$

where $i = \{CA, FR, DE, IT, JP, UK, US\}$, the first equation follows a random walk with stochastic drift terms. These stochastic drift terms each further follow a random walk. The remaining state variables (the common, regional, and idiosyncratic components of the preference shifter) also follow random walks.

The error terms of the measurement and state equations follow normal distributions and they are mutually uncorrelated within a country and are uncorrelated across countries. Thus, the channels through which shocks transmit to other economies are either through regional and common output growth or the preference shifters. This specification enables the decomposition of the natural rate into common, regional, and idiosyncratic components. The complete specification of the model can be found in Appendix B.

3.2 Estimation

I estimate the parameters using a random walk Metropolis-Hastings algorithm. I take the Bayesian approach over the frequentist approach in order to circumvent the pile-up problem that arises in Maximum Likelihood (Stock and Watson, 1998) and to deal with the relatively large dimension of the state space model and the small sample size. Table 1 summarizes my specification of the priors.

I assign the country-by-country estimates in Holston et al. (2017) for the majority of the mean

[9] Appendix A shows how to obtain these results using a simple growth model.

values of the prior distributions.[10] I imposed the prior distribution for the standard deviations of each of the output growths, σ_g, and each of the preference shifters, $\sigma_{z^{common}}$ exploiting the finding in Holston et al. (2017) that the volatility of output growth is smaller than the volatility of the preference shifter.

I make 20,000 draws in total but discard the first 18,000 as a burn-in period. After the burn-in period, I take the remaining draws. From the 2,000 samples, I take the median, 10^{th} %, and 90^{th} % of the state variables from the Kalman Filter. In total, there are 76 parameters and 61 state variables to be estimated. A detailed explanation of the Bayesian estimation is in Appendix C.

4 Results

4.1 r^* in the G7 countries

Figure 1 presents estimates of the natural rate for the G7 countries. The estimates in the figure are the sum of the common, regional (excluding Japan), and idiosyncratic components for each country. The black solid lines represent the median draws from the Kalman filter. The shaded region represents the 10^{th} % to 90^{th} % estimation interval. The posterior parameter draws are reported in Appendix E.1. Additionally, the vertical shaded regions represent recession dates indicated by the National Bureau of Economic Research in the US and the Economic Cycle Research Institutes for the other countries.

The estimates of the natural rate are clearly time varying. The movement in the natural rate are pro-cyclical: when economies enter recessions, r^* decreases, and when the economies expand, r^* increases. This finding implies that when an economy enters a recession, the policy rate needs to be lower to be consistent with potential output, and vice versa. This is also seen in Holston et al. (2017). In the figure, there are two notable drops in the natural rate: the energy crisis during 1980-1990 and the financial crisis in 2008. During these periods, the natural rates of all of the G7 countries fell below zero (excluding Japan during the energy crisis). It is possible that the very low values during 1980s are driven by the âend-point problemâ of the Kalman filter (e.g. Cotis et al., 2004), but, the low values during the financial crisis certainly indicate the strong pro-cyclical nature of the natural rate.

Since the 1990s, the majority of the countries show a tendency for the natural rate to decline.

[10]It is the mode, in the case of the Inverse-Gamma

Table 2 reports the average values of the natural rates for each decade in the sample and the differences of these values between two decades. The natural rate in Canada, Japan, and the US is around 2.5-3% in the 1980s and around 0% in the 2010s. The countries in Europe show a somewhat different path: the natural rate of these countries are below 1% in the 1980s, increase to 1.5-2% in the 1990s and 2000s, and then decrease to similar values as the other countries in the 2010s. This indicates the role of the regional component in the natural rate. However, the low values of the natural rate in the countries in Europe could be due to the end-point problem as discussed above. Overall, for most of the countries, a tendency for r^* to decline is seen since the 1990s.

4.2 Individual Level Estimates

The joint estimates of the natural rate tend to be more volatile than the individual level estimates. Figure 2 shows the country-by-county estimates of the natural rate from Figure 1. The red solid line represents the median joint estimates and the blue solid line represents the median individual estimates. The individual estimates are obtained using a Bayesian method imposing the prior distribution in Appendix Table 12. The joint estimates of the natural rate follow the individual level estimates well, however, the volatility of the joint estimates tend to be higher in some countries, such as countries in Europe. These observations are likely to be driven by the higher volatility of the preference shifter when jointly estimated.[11]

Further, the individual level estimates tend to be higher than the joint estimates of the natural rate. This finding is not only observed during recessions, such as the large drops during the energy crisis and financial crisis periods, but also during the expansion phases. To clearly show this finding, Appendix Figure 12 reports the differences of the median estimates of the natural rate between joint and individual level estimates. Most of the time, the series lies below zero and the average of the series from the G7 countries is -0.5%, indicating the lower natural rate in the joint estimates. Similarly, Kiley (2019) considers a joint estimation and finds r^* estimates are actually lower than the estimates found in the literature. It is possible that single country estimations miss country interactions and spillover effects that propagate shocks across countries and lower r^*.

[11] For example, the standard deviation of the preference shifter of the individual estimate of France is 0.12 (not reported), while the joint estimate of France is 0.2. In addition, the standard deviation of the preference shifter of common and regional components also affects the volatility of the series.

4.3 Decomposition and Synchronization

In the previous section, the estimates of the natural rate for the G7 countries show a tendency to decline since the 1990s. I now investigate which component(s) plays a role in the decline of the natural rate.

The left panel of Figure 3 exhibits the common component of r^*. As before, the black solid line represents the median draw from the Kalman filter. The shaded region represents the 10^{th} % to 90^{th} % estimation interval. The common component captures the main trend of r^* that are seen for all of the G7 countries. The common component displays a declining movement over the sample period. The value is around 0% in the beginning of the sample and is -3% at the end of the sample. This series also demonstrates two large drops during the energy and financial crisis.

The right two panels of Figure 3 show the regional components of r^*. Both the North America and Europe components show drops in the beginning of the sample, increase towards 2000, and then decline towards the end of the sample periods. The large increase in 2000 in the North America component is seen in the natural rate of Canada and the US. I find that the natural rate of some countries in Europe do not show a decline until the 2000s. This is largely due to the Europe component having a large drop at the beginning of the sample and a sharp rise soon after. This indicates a relevant role of the regional component[12] of the natural rate that is not captured in the common component.

Next, Figure 4 shows the idiosyncratic component of r^*. Most of the countries have an erratic and relatively constant series. Note, the declining series in Italy and increasing series in Canada and Germany. Though not always observed, the idiosyncratic components rise during expansions and drop during recessions. In general, the magnitudes of the movements of the idiosyncratic components are smaller than the magnitudes of the movements of the common and regional components.

Decomposing the natural rate itself does not provide the relative importance of each component. To understand whether the natural rate is integrated across the investigated countries, I calculate simple correlations of different components of r^*. Tables 3 and 4 show the correlations of country-level r^*s and idiosyncratic components of country-level r^*s across the G7 countries.

The correlations of the idiosyncratic components of the natural rate range from -74% to 57% (the average is -11%) while the correlations of the country level estimates of the natural rate range

[12] The correlations of r^* in Germany, France, and Italy increase from 82% to 93% since these countries became under the ECB system in 1999. This might explain the non-negligible influence of the regional component.

from 8% to 89% (the average is 63%). The correlations of the idiosyncratic components range from mostly negative values to some moderate positive values, however, the overall correlations of the country level r^*s are mostly above 50%. The idiosyncratic component is a part of the country level r^*. This finding indicates that the comovement of country level r^*s across the G7 countries are not disrupted by the idiosyncratic components, suggesting a synchronization of r^*.

4.4 Output Gap

The estimation method yields a measurement of the output gap at the same time as the measurement of r^*. The IS equation in equation (1) implies that there is a negative relationship between the output gap and the real interest rate gap (real interest rate minus the natural rate).[13] The negative relationship is shown in Figure 5. The solid curves measured on the left axes represent the output gaps and the dotted curves measured on the right axes represent the real interest rate gaps.

Before the 2000s, both variables show clear trends in most countries in Europe: the output gap is persistently below zero and the real interest gap is above zero. This indicates that these economies operated below their potential level during and after the energy crisis, and thus the real interest rates needed to be lower than the rate actually was. On the contrary, Japan, Germany, and the US quickly revert to zero after the large drops in the beginning of the period, suggesting the monetary policy was conducted optimally. After the 2000s, the output gap generally cluster around zero: the deviation of output from its potential and deviation of real interest rate from the natural rate are erratic. This could indicate improvements in monetary policy implementation.

5 Robustness Analysis

In this section, I conduct three robustness analyses. First, I estimate r^* for three countries (Japan, the UK, and the US) which have a longer time series for my sample than in the benchmark data. Second, I estimate r^* for the G7 countries excluding two episodes, the energy crisis and the financial crisis. Third, I estimate r^* by modifying the assumption that the preference shifters follow random walks.

[13] Figure 13 displays both the r^* estimates and the real interest rate for each country.

5.1 Three Country Estimation

I estimate a three-country (Japan, the UK, and the US) state space model with a that spans from 1962Q1-2019Q4. I use the same three variables: real GDP, the core CPI (core PCE for the US), and the monetary policy rate. The specification of the decomposition is the following:

$$r^*_{i,t} = \quad r^{*common}_t + r^{*idiosyncratic}_{i,t} \tag{4}$$

The natural rate is decomposed into common and idiosyncratic components. The regional component is excluded in this analysis because the countries are distantly located and it would be infeasible to differentiate the regional component from the idiosyncratic component if the regional component were included. Figure 6 shows the estimates of the natural rate by estimating the model jointly for the three countries. Similar to the benchmark estimates, the series demonstrates moderate volatility, pro-cyclical movements, and downward trends. The natural rate declines faster in Japan than in the UK and the US. Eventually the estimates are around 1% at the end of the sample.

Figure 7 represents the decomposition of the country estimates of r^*. Akin to the finding in the benchmark exercise, the common component from the three countries show a persistent decline in r^*. The movement of the idiosyncratic components are persistently declining in Japan, rising in the UK, and erratic in the US.

Table 5 shows the correlations of r^*s and the idiosyncratic components of r^*s. As seen in the previous section, the correlations of r^*s between countries are high, ranging from 63% to 84% (the average is 76%). However, the correlations of the idiosyncratic components across countries range from -94% to 37% (the average is -38%). The idiosyncratic components do not seem to have much of an effect on the comovements of country level r^*s, indicating the synchronization of r^*s across the three countries.

5.2 Excluding the Energy Crisis and Financial Crisis

In the benchmark results, the estimated r^* shows relatively large swings during the energy crisis period and the financial crisis period. The behavior of r^* during these periods might be as outliers, and excluding these periods might dramatically alter the estimated parameters and r^*.

In this section I exclude the energy crisis period, which makes the sample period 1984Q2-2019Q, and seperately the financial crisis period, which makes the sample period 1979Q4-2007Q4. The specification and estimation methods are the same as the benchmark methodology.

Figure 8 shows the estimated natural rate for the G7 countries excluding the energy crisis and the financial crisis periods. To avoid a crowded figure, I only report the median series: the solid line represents the benchmark median r^*, the dashed line represents median r^* excluding the financial crisis period, and the dotted line represents median r^* excluding the energy crisis period.

The exclusion of the energy crisis makes the estimates of the natural rate in the countries of Europe higher than the benchmark estimates during 1985-1990. The exclusion of the financial crisis does not generate a persistent pattern of deviation from the benchmark result. Overall, r^* is very similar to the benchmark r^*.

Figure 9 plots the common and regional components of r^* excluding the energy crisis and the financial crisis. Some deviations from the benchmark estimates appear. After 1990 the exclusion of these two episodes makes the common component higher than the benchmark estimates. The regional component excluding the energy crisis makes the series persistently lower than the benchmark regional component. The component excluding the financial crisis is comparable to the benchmark. Overall, the dynamics of the components of r^* are similar to the dynamics of the benchmark series.

With regard to the synchronization of the natural rate across the G7 countries, Tables 6, 7, 8, and 9 report the correlations of the natural rate and the idiosyncratic component of the natural rate with different subsample periods.

Looking at the exclusion of the financial crisis period (Tables 6 and 7), the range of the correlations of r^* is -19% to 82% (the average is 45%) and the range of the correlations of the idiosyncratic components of r^* is -80% to 78% (the average is -13%). Compared to the benchmark results, the average and range of the correlations decline slightly. Thus, the synchronization of r^* becomes weaker than in the benchmark results, even though most of the correlations of r^* are still higher than the correlations of the idiosyncratic components. This finding is explained by two possibilities. The first is the smaller sample size. I drop 30% of the sample size compared to the benchmark and this might reduce the estimation accuracy. A second possibility is that the synchronization accelerated after the financial crisis as shown in Del Negro et al., 2019, due to the higher demand for safe and liquid assets.

Looking at the exclusion of the energy crisis, (Tables 8 and 9), the range of the correlation of r^* is 55% to 93% (the average is 78%) and the range of the correlations of the idiosyncratic components of r^* is -69% to 60% (the average is -10%). This indicates that the idiosyncratic component of r^* does not play a major role in the dynamics of r^* across the G7 countries. Thus, excluding the energy crisis does not distort the findings in Section 4 and supports the synchronization of r^* across the G7 countries. Excluding the energy crisis actually increases the correlations of r^* compared to the correlations of the benchmark r^*. This also supports the notion that the synchronization accelerated after the financial crisis.

5.3 Stationary Processes on the Preference Shifters

The preference shifter, z_t, is driven from the time preference parameter, ρ, in Appendix A. The benchmark specification assumes that the preference shifter, z_t, follows a random walk as in Holston et al. (2017). However, it is also reasonably possible to interpret the time preference as a constant. In this section, I impose an assumption that the preference shifter is stationary and follows an autoregressive (AR) process in the spirit of Lewis and Vazquez-Grande (2017).

The benchmark specification assumes that the natural rate consists of three different components of the preference shifter, z_t. It is possible that some series follow random walk processes and other series follow AR processes. One could test all of the possible combinations of those processes. However, for the sake of brevity, I will assume that all of the components follow AR processes. This enables the investigation of two extreme specifications: the benchmark specification with the preference shifters following random walks, and the estimation here that the preference shifters follow AR processes. I assume that each preference shifter follows an AR(1) process, which increases the number of parameters to be estimated. The prior specification of the AR coefficients (and the standard deviations of the preference shifter) is available in Appendix Table 13.

Figure 10 shows the estimated natural rate of the G7 countries assuming that the preference shifters follow AR processes. The red solid line represents the benchmark estimates and the blue solid line represents the estimates in this specification. The results are generally similar to the benchmark series. However, this specification makes the series slightly more volatile and slightly higher than the benchmark estimates.

A notable difference due to this specification appears in the regional components of the natural rates. Figure 11 reports the decompositions of the natural rate. The common and North America components in this specification follow the benchmark series reasonably well. However, the Europe component is noteworthy: the series is clustered around 0.2 and the volatility is vastly smaller than the benchmark Europe component. The differences between the benchmark and this specification suggest that the preference shifter is an important driver of the regional component in Europe but not in North America. Given the higher volatility of the preference shifter in this paper and in Holston et al. (2017) in the Europe area, it seems a random walk specification fits better. To sum up, the estimated r^* does not seem to be largely affected by this specification and this exercise reveals the importance of the random walk specification for the countries in Europe.

6 Conclusion

In this paper, I estimate a multi-country state space model and jointly estimate the natural rate for the G7 countries. I then decompose the variable into common, regional, and idiosyncratic components. I find that the common component is the main driver of r^*, not the idiosyncratic component. Also, I find that r^* is steadily declining since the 1990s and the estimates are lower than the estimates found in literature that does not consider the international component.

These findings are calculated using the G7 countries which are highly advanced economies. These countries have similar characteristics such as demographics and productivity[14], which are typical determinants of r^* in the literature[15]. Thus, the implications of this paper may not apply to developing countries and emerging market economies. Looking into the decomposition of r^* in those countries would be a great topic in future research.

[14]For examples, Tuljapurkar et al. (2000) for demographics and OECD (2017) for productivity

[15]Gillman, 2021 and Gillman and Csabafi, 2021 show an interesting view of low real rate due to an implicit tax on capital market.

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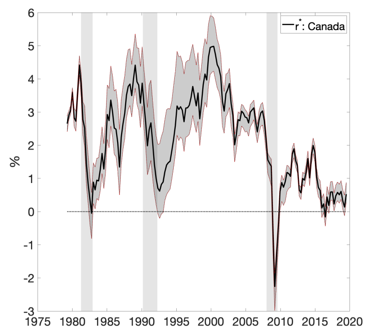
7 Figures

Figure 1: The Natural Rate of Interest



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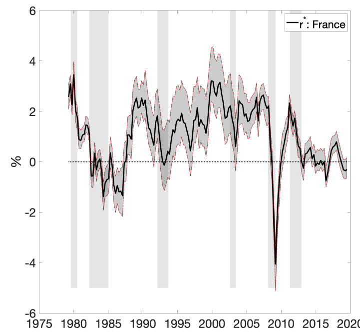

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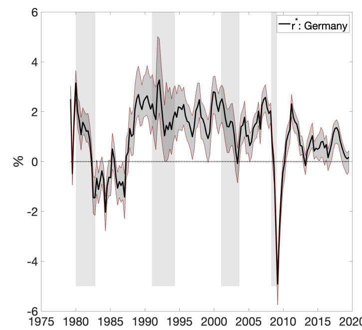

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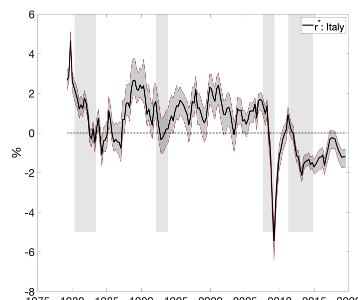

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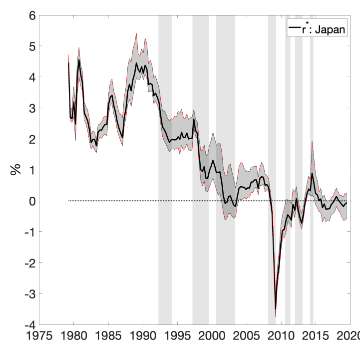

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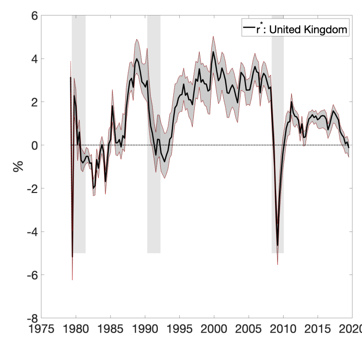

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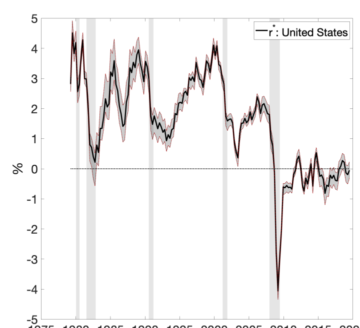

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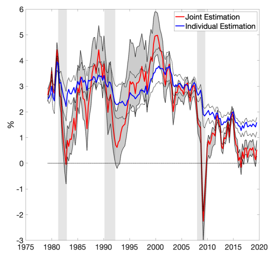
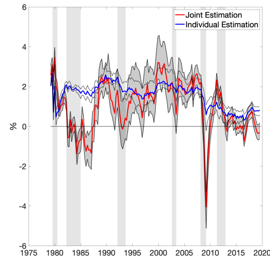
Figure 2: The Natural Rate of Interest: Joint Estimates vs Country-by-Country Estimates



(a) Canada



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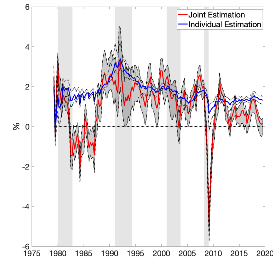

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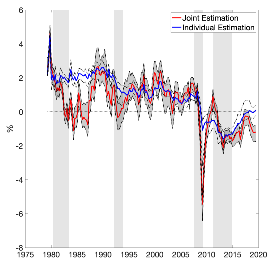

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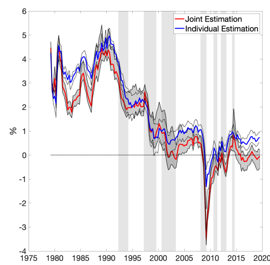

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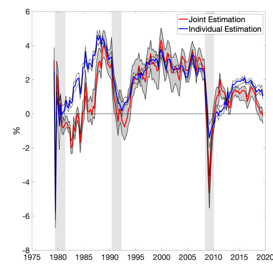

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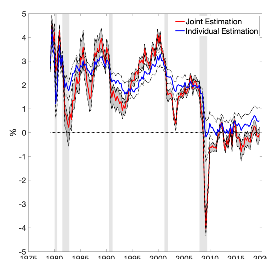

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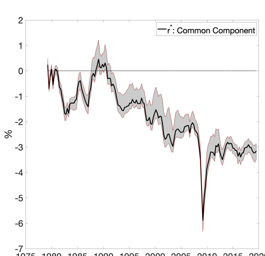
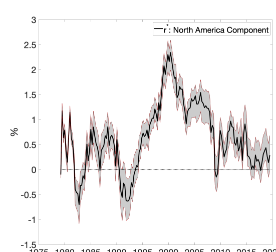
Figure 3: The Common Component and Regional Component of r^*



(a) Common Component



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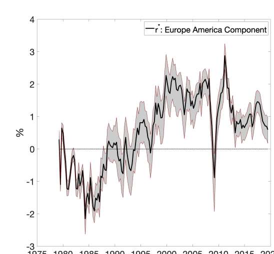
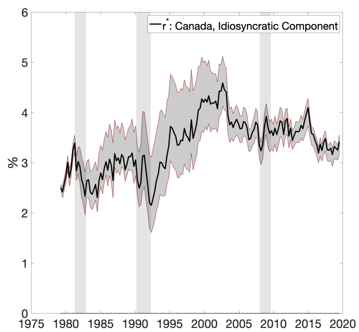
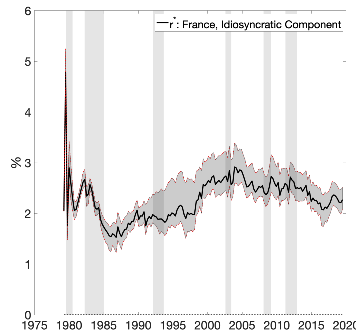


(c) Europe Component

Figure 4: The Idiosyncratic Component of r^*



(a) Idiosyncratic (Canada)



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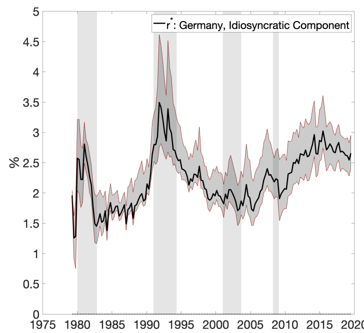
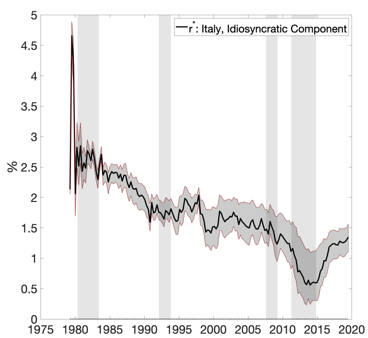
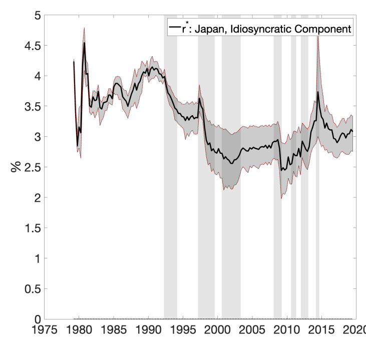


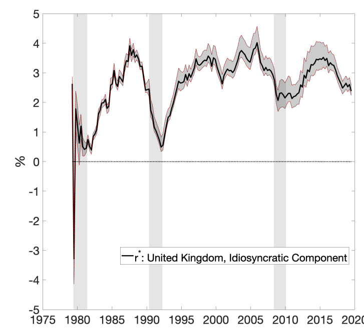
(c) Idiosyncratic (Germany)



(d) Idiosyncratic (Italy)



(e) Idiosyncratic (Japan)



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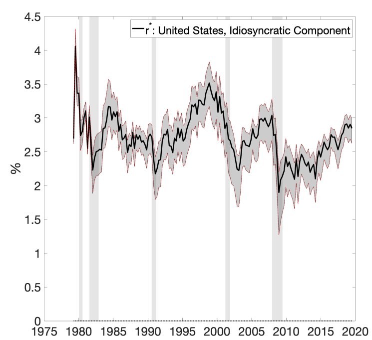
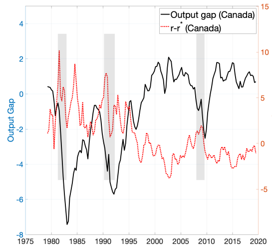
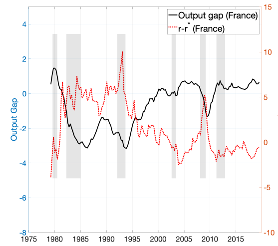


(g) Idiosyncratic (US)

Figure 5: Output Gap and Real Interest Rate Gap



(a) Canada



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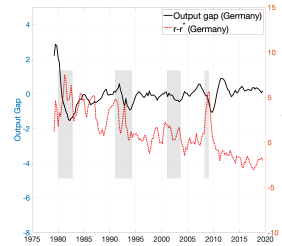

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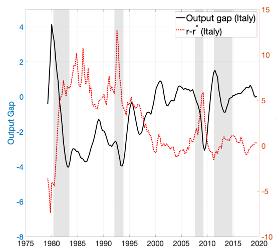

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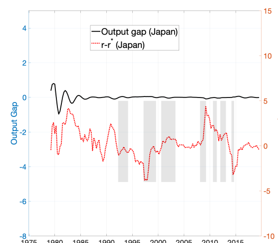

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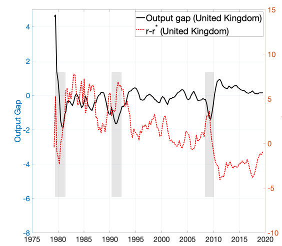

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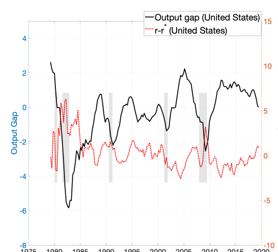
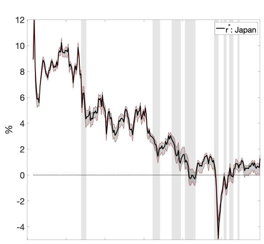
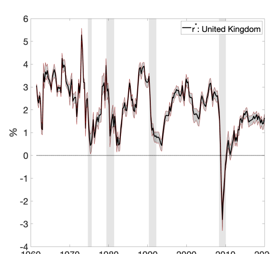


(g) United States

Figure 6: The Natural Rate of Interest (Three Country Estimation)



(a) Japan



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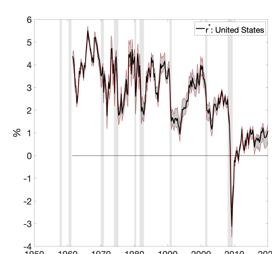
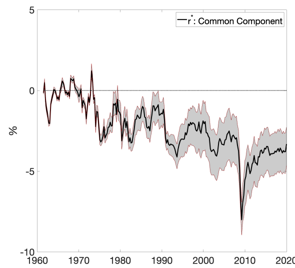
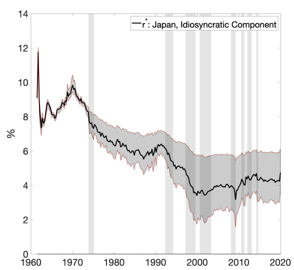


(c) United States

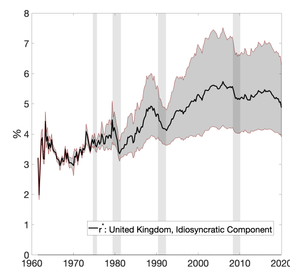
Figure 7: The Natural Rate of Interest Decomposition (Three Country Estimation)



(a) Common Component



(b) Idiosyncratic (Japan)



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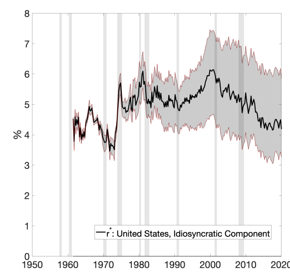

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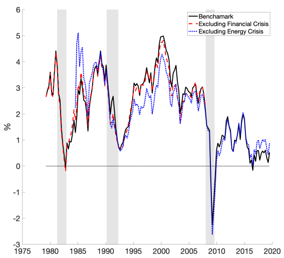
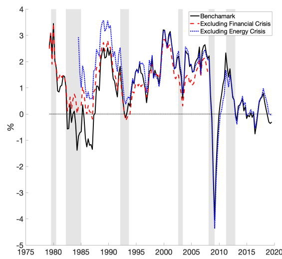
Figure 8: The Natural Rate of Interest (Sub-Sample Estimates)



(a) Canada



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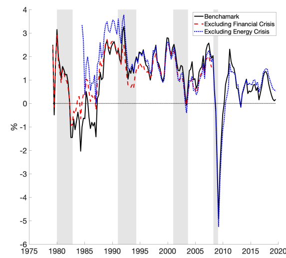

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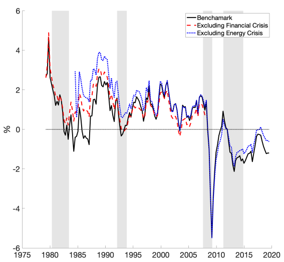

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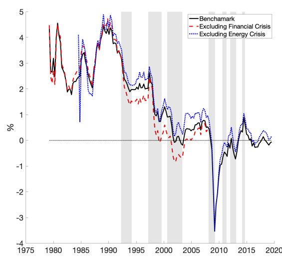

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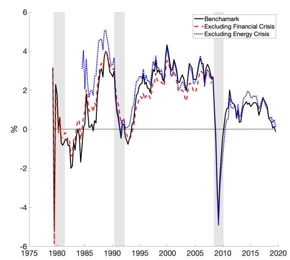

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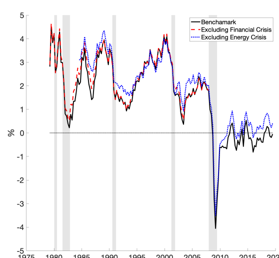
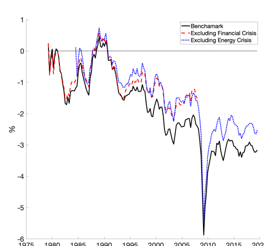


(g) United States

Figure 9: Common Component and Regional Component of r^* (Sub-Sample Estimates)



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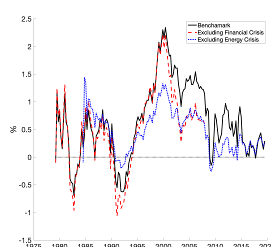

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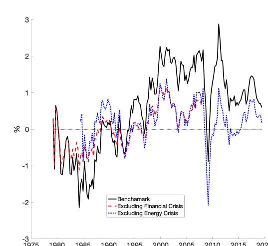
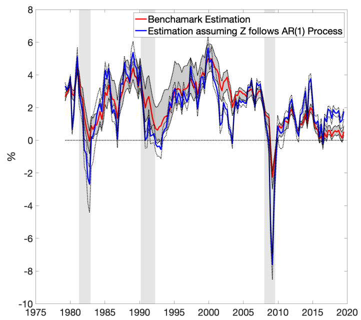
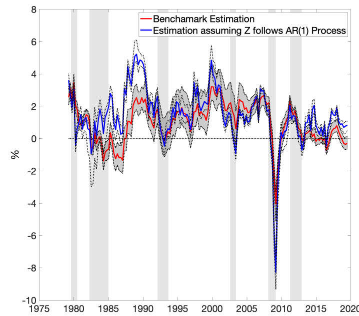


(c) Europe Component

Figure 10: The Natural Rate of Interest (Stationary Processes on the Preference Shifter)



(a) Canada



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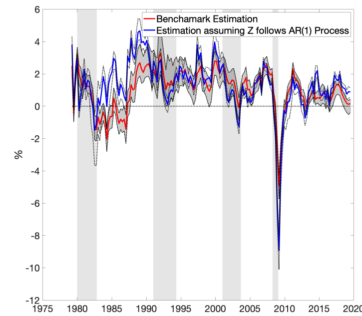

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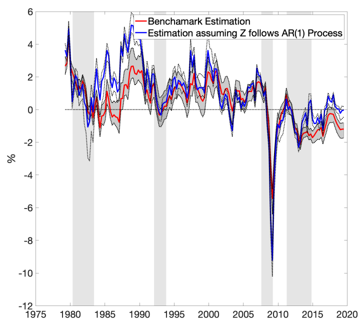

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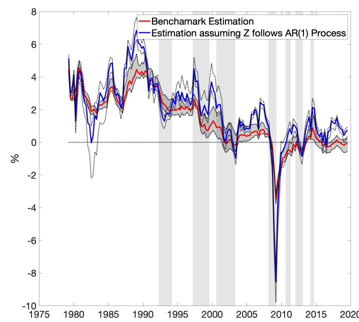
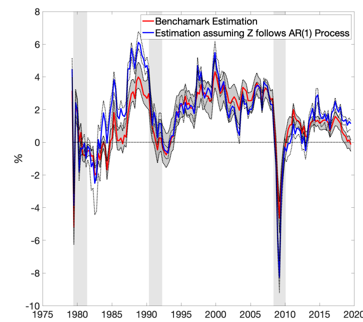
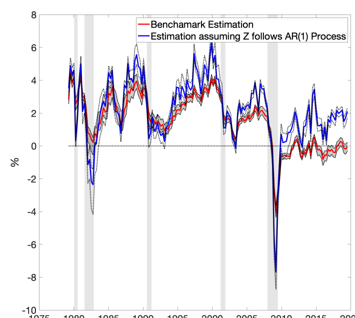


(e) Japan

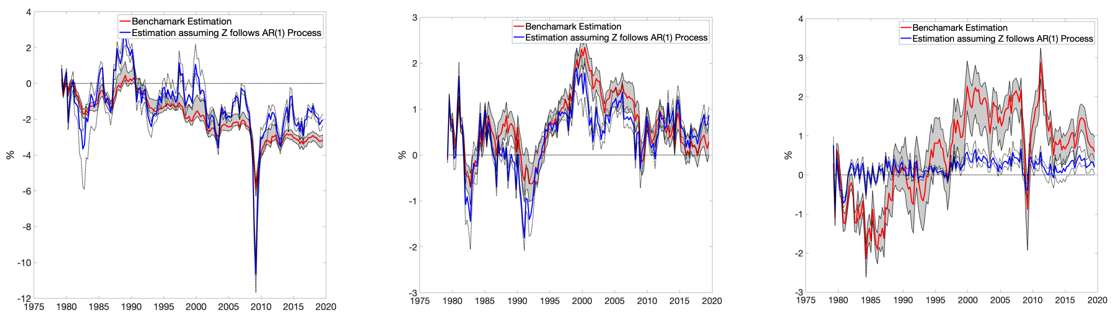


(f) United Kingdom



(g) United States

Figure 11: The Natural Rate of Interest Decomposition (Stationary Processes on the Preference Shifter)



(a) Common Component (b) North America Component (c) Europe Component

8 Tables

Table 1: Prior Specification

Parameter	Domain	Density	Parameter1	Parameter2
$a_{y1,i}$	$[0,\infty)$	Normal	MLE	0.3
$a_{y2,i}$	R	Normal	MLE	0.3
$a_{r,i}$	$(-\infty,-0.0025)$	Normal	min(MLE,-0.01)	0.3
$b_{\pi,i}$	[0,1]	Beta	MLE	0.3
$b_{y,i}$	$(0.025,\ \infty)$	Normal	min(MLE,0.5)	0.3
σ_{y_i}	$(0,\infty)$	Inverse-Gamma	MLE	0.1
σ_{π_i}	$(0,\infty)$	Inverse-Gamma	MLE	0.1
$\sigma_{y_i^*}$	$(0,\infty)$	Inverse-Gamma	MLE	0.1
σ_{g_i}	$(0,\sigma_{z_i})$	Inverse-Gamma	0.03	0.03
σ_{z_i}	$(0,\infty)$	Inverse-Gamma	0.1	0.1
$\sigma_{g^{common}}$	(0,0.05)	Inverse-Gamma	0.03	0.03
$\sigma_{z^{common}}$	$(0,\ \infty)$	Inverse-Gamma	0.1	0.1
$\sigma_{g^{north-america}}$	$(0,\ \sigma_{z^{north-america}})$	Inverse-Gamma	0.03	0.03
$\sigma_{z^{north-america}}$	$(0,\infty)$	Inverse-Gamma	0.1	0.1
$\sigma_{g^{europe}}$	$(0,\ \sigma_{z^{europe}})$	Inverse-Gamma	0.03	0.03
$\sigma_{z^{europe}}$	$(0,\infty)$	Inverse-Gamma	0.1	0.1

: $i = \{CA, FR, DE, IT, JP, UK, US\}$. Parameter 1 is the mean of the normal distribution, the mean of the beta distribution, and the mode value of the inverse-gamma distribution. Parameter 2 is the standard deviation of the all of the distributions. MLE represents the estimates from the maximum likelihood on a country-by-country basis using the three-step estimation method in Holston et al. 2017. Since some of the Maximum Likelihood estimates on the slope parameters of the IS curve, $a_{r,i}$, and the Phillips curve, $b_{y,i}$, were very different from the estimates found in Holston et al. (2017), I imposed a constraint on the prior mean of the standard deviation of the common growth trend, $\sigma_{g^{common}}$ to avoid a large decline in the natural rate during the financial crisis (e.g. The median value of the natural rate is -10% in Japan), though qualitatively there is no difference in the results.

Table 2: The Natural Rate Estimates of the G7 Countries

	Canada	France	Germany	Italy	Japan	United Kingdom	United States
1980s	2.55	0.65	0.47	1.03	3.04	0.62	2.53
1990s	2.66	1.33	1.89	1.05	2.32	1.72	2.38
2000s	2.59	1.56	1.04	0.58	0.10	2.16	1.31
2010s	0.86	0.41	0.77	-0.91	-0.14	0.99	-0.22
Change							
1990s - 1980s	0.11	0.67	1.41	0.02	-0.72	1.10	-0.16
2000s - 1990s	-0.07	0.23	-0.85	-0.47	-2.22	0.44	-1.07
2010s - 2000s	-1.73	-1.15	-0.27	-1.49	-0.24	-1.17	-1.53

The first four rows show the averages of the natural rate of interest for each decade in the sample. The bottom three numbers are the differences of those numbers.

Table 3: Correlations of the Natural Rates in the G7 Countries

	r^*_{CA}	r^*_{FR}	r^*_{DE}	r^*_{IT}	r^*_{JP}	r^*_{UK}	r^*_{US}
r^*_{CA}	1						
r^*_{FR}	0.71	1					
r^*_{DE}	0.59	0.87	1				
r^*_{IT}	0.79	0.85	0.75	1			
r^*_{JP}	0.49	0.24	0.36	0.63	1		
r^*_{UK}	0.67	0.71	0.70	0.57	0.08	1	
r^*_{US}	0.89	0.59	0.53	0.84	0.74	0.48	1

The correlations are calculated based on the median estimates of the natural rate of interest.

Table 4: Correlations of Idiosyncratic Component of Natural Rates in the G7 Countries

	r^{*idio}_{CA}	r^{*idio}_{FR}	r^{*idio}_{DE}	r^{*idio}_{IT}	r^{*idio}_{JP}	r^{*idio}_{UK}	r^{*idio}_{US}
r^{*idio}_{CA}	1						
r^{*idio}_{FR}	0.47	1					
r^{*idio}_{DE}	-0.03	-0.12	1				
r^{*idio}_{IT}	-0.56	-0.16	-0.56	1			
r^{*idio}_{JP}	-0.74	-0.63	0.07	0.48	1		
r^{*idio}_{UK}	0.57	-0.14	-0.17	-0.52	-0.39	1	
r^{*idio}_{US}	-0.12	-0.01	-0.28	0.42	0.07	-0.01	1

The correlations are calculated based on the median estimates of the natural rate of interest.

Table 5: Correlations of Natural Rates and Idiosyncratic Components of Natural Rates in Japan, UK, and US

	r^*_{JP}	r^*_{UK}	r^*_{US}	r^{*idio}_{JP}	r^{*idio}_{UK}	r^{*idio}_{US}
r^*_{JP}	1					
r^*_{UK}	0.63	1				
r^*_{US}	0.80	0.84	1			
r^{*idio}_{JP}	0.95	0.38	0.61	1		
r^{*idio}_{UK}	-0.88	-0.22	-0.60	-0.94	1	
r^{*idio}_{US}	-0.47	-0.11	0.06	-0.58	0.37	1

The correlations are calculated based on the median estimates of the natural rate of interest.

Table 6: Correlations of r^*s in the G7 Countries Excluding the Financial Crisis

	r^*_{CA}	r^*_{FR}	r^*_{DE}	r^*_{IT}	r^*_{JP}	r^*_{UK}	r^*_{US}
r^*_{CA}	1						
r^*_{FR}	0.66	1					
r^*_{DE}	0.55	0.78	1				
r^*_{IT}	0.47	0.71	0.62	1			
r^*_{JP}	-0.10	0.03	0.20	0.58	1		
r^*_{UK}	0.64	0.50	0.62	0.23	-0.19	1	
r^*_{US}	0.82	0.57	0.48	0.70	0.32	0.32	1

The correlations are calculated based on the median estimates of the natural rate of interest.

Table 7: Correlations of r^{*idio}s in the G7 Countries Excluding the Financial Crisis

	r^{*idio}_{CA}	r^{*idio}_{FR}	r^{*idio}_{DE}	r^{*idio}_{IT}	r^{*idio}_{JP}	r^{*idio}_{UK}	r^{*idio}_{US}
r^{*idio}_{CA}	1						
r^{*idio}_{FR}	0.31	1					
r^{*idio}_{DE}	0.03	-0.07	1				
r^{*idio}_{IT}	-0.58	0.09	-0.44	1			
r^{*idio}_{JP}	-0.80	-0.46	-0.09	0.54	1		
r^{*idio}_{UK}	0.66	-0.27	-0.10	-0.73	-0.54	1	
r^{*idio}_{US}	-0.72	0.05	-0.22	0.78	0.49	-0.69	1

The correlations are calculated based on the median estimates of the natural rate of interest.

Table 8: Correlations of r^*s in the G7 Countries Excluding the Energy Crisis

	r^*_{CA}	r^*_{FR}	r^*_{DE}	r^*_{IT}	r^*_{JP}	r^*_{UK}	r^*_{US}
r^*_{CA}	1						
r^*_{FR}	0.78	1					
r^*_{DE}	0.59	0.85	1				
r^*_{IT}	0.80	0.91	0.86	1			
r^*_{JP}	0.61	0.70	0.82	0.87	1		
r^*_{UK}	0.85	0.81	0.66	0.77	0.55	1	
r^*_{US}	0.91	0.83	0.74	0.93	0.80	0.82	1

The correlations are calculated based on the median estimates of the natural rate of interest.

Table 9: Correlations of r^{*idio}s in the G7 Countries Excluding the Energy Crisis

	r^{*idio}_{CA}	r^{*idio}_{FR}	r^{*idio}_{DE}	r^{*idio}_{IT}	r^{*idio}_{JP}	r^{*idio}_{UK}	r^{*idio}_{US}
r^{*idio}_{CA}	1						
r^{*idio}_{FR}	0.49	1					
r^{*idio}_{DE}	-0.40	-0.14	1				
r^{*idio}_{IT}	-0.34	-0.50	-0.53	1			
r^{*idio}_{JP}	-0.69	-0.63	0.19	0.53	1		
r^{*idio}_{UK}	0.60	0.13	-0.39	-0.20	-0.45	1	
r^{*idio}_{US}	0.09	-0.15	-0.56	0.44	-0.02	0.34	1

The correlations are calculated based on the median estimates of the natural rate of interest.

A A Simple Model Explaining the Link Between Output Growth and Real Interest Rate

Consider an economy where technology A_t and labor L_t grow at rates g_A and n. Production uses labor and capital, and the production function is labor-augmenting and constant returns to scale. The representative consumer's optimal intertemporal consumption path or Eular equation is:

$$u^{'}(c_t) = \frac{1}{1+\rho}u^{'}(c_{t+1})(F_K(K_t, A_tL_t) + 1) \tag{5}$$

where $u^{'}(c_t) = \dfrac{du(c_t)}{dc_{c_t}}$, $\dfrac{1}{1+\rho}$ represents time preference with $0 < \dfrac{1}{1+\rho} < 1$, and $F_K(K_t, A_tL_t) = \dfrac{\partial F(K_t, A_tL_t)}{\partial K_t} = r_t$.

Here, the utility function takes a form of constant relative risk aversion (CRRA):

$$u(c_t) = \frac{c_t^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} \tag{6}$$

Then equation (5) becomes:

$$\left(\frac{c_{t+1}}{c_t}\right)^{\frac{1}{\sigma}} = \frac{1}{1+\rho}(r_t + 1) \tag{7}$$

Take logarithm on both sides:

$$\frac{1}{\sigma}\ln\left(\frac{c_{t+1}}{c_t}\right) = \ln(r_t + 1) - \ln(1+\rho)$$

Using the first order Taylor approximation:

$$\frac{1}{\sigma}\left(\frac{c_{t+1} - c_t}{c_t}\right) \approx r_t - \rho \tag{8}$$

using the fact that $\ln\left(\frac{x_{t+1}}{x_t}\right) \approx \frac{x_{t+1} - x_t}{x_t}$ and $\ln(x_t + 1) \approx x_t$.

Under a balanced growth path and from equation (8), we obtain:

$$r^* = \sigma^{-1} g_A + \rho$$

This is the equation in Holston et al. (2017) that summarizes the one-to-one relationship of output growth and real interest rate.

B A Complete Description of the State Space Model

B.0.1 Measurement Equations

B.1 Measurement Equations

$$Y_t = AX_t + HM_t + \nu_t$$

or

$$\begin{bmatrix} Y_{CA,t} \\ \vdots \\ Y_{US,t} \end{bmatrix} = \begin{bmatrix} A_{CA} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & A_{US} \end{bmatrix} \begin{bmatrix} x_{CA,t} \\ \vdots \\ x_{US,t} \end{bmatrix} + \begin{bmatrix} H_{CA} & & \mathbf{0} & -\frac{a_{r,CA}}{2}_{1\times4} & \\ & \ddots & & \vdots & \mathrm{h} \\ \mathbf{0} & & H_{US} & -\frac{a_{r,US}}{2}_{1\times4} & \end{bmatrix} M_t + \begin{bmatrix} \nu_{CA,t} \\ \vdots \\ \nu_{US,t} \end{bmatrix}$$

where

$$Y_{i,t} = \begin{bmatrix} y_{i,t} \\ \pi_{i,t} \end{bmatrix}, \quad A_i = \begin{bmatrix} a_{y1,i} & a_{y2,i} & \frac{a_{r,i}}{2} & \frac{a_{r,i}}{2} & 0 & 0 \\ b_{y,i} & 0 & 0 & 0 & b_{\pi,i} & 1-b_{\pi,i} \end{bmatrix}, \quad x_{i,t} = \begin{bmatrix} y_{i,t-1} \\ y_{i,t-1} \\ r_{i,t-1} \\ r_{i,t-2} \\ \pi_{i,t-1} \\ \pi_{i,t-2,4} \end{bmatrix}$$

$$H_i = \begin{bmatrix} 1 & -a_{y1,i} & -a_{y2,i} & -\frac{a_{r,i}}{2} & -\frac{a_{r,i}}{2} & -\frac{a_{r,i}}{2} & -\frac{a_{r,i}}{2} \\ 0 & -b_{y,i} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathrm{h} = \begin{bmatrix} -\frac{a_{r,CA}}{2}_{1\times4} & 0_{1\times4} \\ 0_{1\times4} & 0_{1\times4} \\ 0_{1\times4} & -\frac{a_{r,FR}}{2}_{1\times4} \\ 0_{1\times4} & 0_{1\times4} \\ 0_{1\times4} & -\frac{a_{r,DE}}{2}_{1\times4} \\ 0_{1\times4} & 0_{1\times4} \\ 0_{1\times4} & -\frac{a_{r,IT}}{2}_{1\times4} \\ 0_{1\times4} & 0_{1\times4} \\ 0_{1\times4} & 0_{1\times4} \\ 0_{1\times4} & 0_{1\times4} \\ 0_{1\times4} & -\frac{a_{r,UK}}{2}_{1\times4} \\ 0_{1\times4} & 0_{1\times4} \\ -\frac{a_{r,US}}{2}_{1\times4} & 0_{1\times4} \\ 0_{1\times4} & 0_{1\times4} \end{bmatrix},$$

$$Mt = \begin{bmatrix} \xi_{CA,t} \\ \vdots \\ \xi_{US,t} \\ g_{t-1}^{common} \\ g_{t-2}^{common} \\ z_{t-1}^{common} \\ z_{t-2}^{common} \\ g_{t-1}^{north-america} \\ g_{t-2}^{north-america} \\ z_{t-1}^{north-america} \\ z_{t-2}^{north-america} \\ g_{t-1}^{europe} \\ g_{t-2}^{europe} \\ z_{t-1}^{europe} \\ z_{t-2}^{europe} \end{bmatrix} \quad \text{with} \quad \xi_{i,t} = \begin{bmatrix} y_{i,t}^{*} \\ y_{i,t-1}^{*} \\ y_{i,t-2}^{*} \\ g_{t-1}^{idiosyncratic} \\ g_{t-2}^{idiosyncratic} \\ z_{t-1}^{idiosyncratic} \\ z_{t-2}^{idiosyncratic} \end{bmatrix}$$

B.1.1 State Equations

$$M_t = FM_{t-1} + \epsilon_t$$

or

$$M_t = \begin{bmatrix} f & & \Big0 & \omega & z_{CA} \\ & \ddots & & \vdots & \vdots \\ \Big0 & & f & \omega & z_{US} \\ \mathbf{0} & \dots & \mathbf{0} & & \delta \end{bmatrix} M_{t-1} + \begin{bmatrix} \epsilon_{CA,t} \\ \vdots \\ \epsilon_{US,t} \\ e_t \end{bmatrix}$$

where

$$f = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \omega = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad z_{CA} = z_{US} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$z_{FR} = z_{DE} = z_{IT} = z_{US} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad z_{JP} = \mathbf{0}_{7\times 8}$$

$$\delta = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad \epsilon_{i,t} = \begin{bmatrix} \epsilon_{t,y_i^*} \\ 0 \\ 0 \\ \epsilon_{t,g_i^{idiosyncratic}} \\ 0 \\ \epsilon_{t,z_i^{idiosyncratic}} \\ 0 \end{bmatrix}, \quad \text{and} \quad e_t = \begin{bmatrix} \epsilon_{t,g^{common}} \\ 0 \\ \epsilon_{t,z^{common}} \\ 0 \\ \epsilon_{t,g^{north-america}} \\ 0 \\ \epsilon_{t,z^{north-america}} \\ 0 \\ \epsilon_{t,g^{europe}} \\ 0 \\ \epsilon_{t,z^{europe}} \\ 0 \end{bmatrix}$$

$$\nu_t \sim \mathcal{N}(\mathbf{0}, R) \qquad \text{where} \qquad R = \begin{bmatrix} R_{CA} & & \Large 0 \\ & \ddots & \\ \Large 0 & & R_{US} \end{bmatrix} \quad \text{with} \quad R_i = \begin{bmatrix} \sigma_{\tilde{y}_i} & 0 \\ 0 & \sigma_{\pi i} \end{bmatrix}$$

$$\epsilon_t \sim \mathcal{N}(\mathbf{0}, Q) \qquad \text{where} \qquad Q = \begin{bmatrix} q_{CA} & & \Large 0 & \mathit{\Omega} & s_{CA} \\ & \ddots & & \vdots & \vdots \\ \Large 0 & & q_{US} & \mathit{\Omega} & s_{US} \\ \mathit{\Omega}^T & \dots & \mathit{\Omega}^T & & \\ s_{CA}^T & \dots & s_{US}^T & & \mathit{\Gamma}_{2\times 2} \end{bmatrix}$$

Here

$$q_{CA} = q_{US} = \begin{bmatrix} \sigma^2_{y^*i} + \sigma^2_{g^{common}} + \sigma^2_{g^{north-america}} + \sigma^2_{g_i} & 0 & 0 & \sigma^2_{g_i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma^2_{g_i} & 0 & 0 & \sigma^2_{g_i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2_{z_i} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$q_{FR} = q_{DE} = q_{IT} = q_{UK} = \begin{bmatrix} \sigma^2_{y^*i} + \sigma^2_{g^{common}} + \sigma^2_{g^{europe}} + \sigma^2_{g_i} & 0 & 0 & \sigma^2_{g_i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma^2_{g_i} & 0 & 0 & \sigma^2_{g_i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2_{z_i} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$q_{JP} = \begin{bmatrix} \sigma^2_{y^*JP} + \sigma^2_{g^{common}} + \sigma^2_{g_i} & 0 & 0 & \sigma^2_{g_{JP}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma^2_{g_{JP}} & 0 & 0 & \sigma^2_{g_{JP}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2_{z_{JP}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Omega = \begin{bmatrix} \sigma^2_{g^{common}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad s_{CA} = s_{US} = \begin{bmatrix} \sigma^2_{g^{north-america}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$s_{FR} = s_{DE} = s_{IT} = s_{UK} = \begin{bmatrix} 0 & 0 & 0 & 0 & \sigma^2_{g^{europe}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad s_{JP} = \mathbf{0}_{8\times 8}, \quad \text{and}$$

$$
\Gamma = \begin{bmatrix}
\sigma^2_{g^{common}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sigma^2_{z^{common}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \sigma^2_{g^{north-america}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \sigma^2_{z^{north-america}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma^2_{g^{europe}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma^2_{z^{europe}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Finally, ν_t and ϵ_t follow Gaussian distribution and are mutually uncorrelated.

C A Detailed Explanation of the Bayesian Estimation

Given the assumption that the error terms of the state space model follow the Gaussian distribution, the density of the data $f(Y_t, X_t|M_t)$ is:

$$f(Y_t, X_t|M_t) = (2\pi)^{-\frac{1}{2}}|f_{t,t-1}|^{-\frac{1}{2}} exp(-0.5u_{t,t-1}f^{-1}_{t,t-1}u_{t,t-1})$$

where $u_{t,t-1}$ is the predictive error and $f_{t,t-1}$ is the variance of the predictive error of the Kalman filter. The likelihood function of the model is:

$$f(Y, X|\theta) = \prod_{t=1}^{T} f(Y_t, X_t|M_t)$$

where $\theta = \{\theta_{CA}, \ldots, \theta_{US}, \sigma_{g^{common}}, \sigma_{z^{common}}, \sigma_{g^{north-america}}, \sigma_{z^{north-america}}, \sigma_{g^{europe}}, \sigma_{z^{europe}}\}$ with $\theta_i = \{a_{y1,i}, a_{y2,i}, a_{r,i}, b_{\pi,i}, b_{y,i}, \sigma_{\tilde{y_i}}, \sigma_{\pi i}, \sigma_{y^* i}, \sigma_{g_i}, \sigma_{z_i}\}$.

I conducted random walk Metropolis-Hastings approach in the following steps:

Step 1: Specify a starting value θ_0 and variance of the shock Σ.

Step 2: Draw a new parameter vector from the random walk equation:

$$\theta_{NEW} = \theta_{OLD} + e \qquad e \sim \mathcal{N}(\mathbf{0}, \Sigma)$$

Step 3: Compute the acceptance probability:

$$\alpha = min\left(\frac{f(Y, X|\theta_{NEW})p(\theta_{NEW})}{f(Y, X|\theta_{OLD})p(\theta_{OLD})}, 1\right)$$

where $p(\theta_i)$ is the prior density.

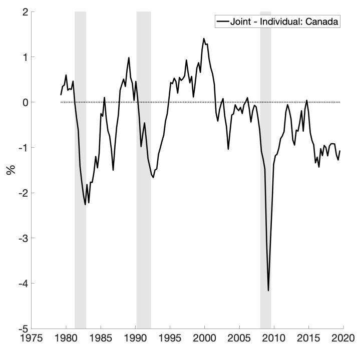
Step 4: If $\alpha > a \sim \mathcal{U}(0, 1)$, obtain θ_{NEW}. Otherwise $\theta_{NEW} = \theta_{OLD}$.

Repeat steps 2, 3, and 4 20,000 times. The first 18,000 draws are a burn-in period and after the burn-in period, I draw 2,000 samples of parameters.

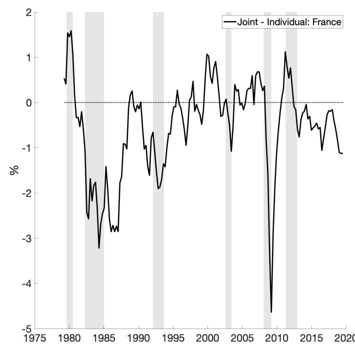
D Appendix: Figures

D.1 Joint Estimate minus Individual Estimate of the Natural rate

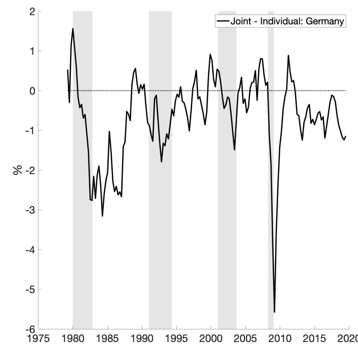
Figure 12: The Natural Rate of Interest: Joint Estimates Minus Country-by-Country Estimates



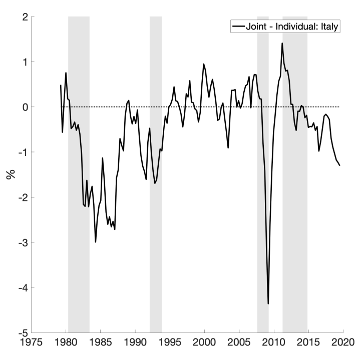
(a) Canada



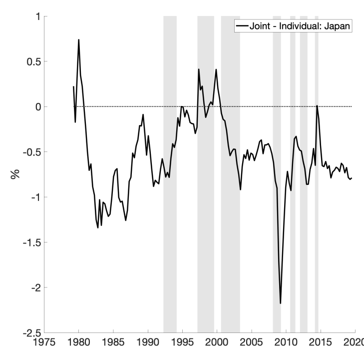
(b) France



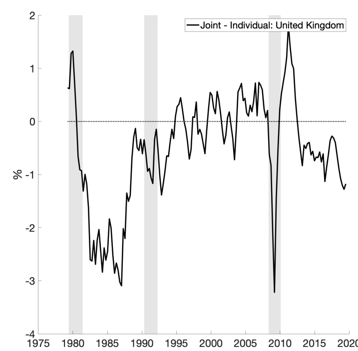
(c) Germany



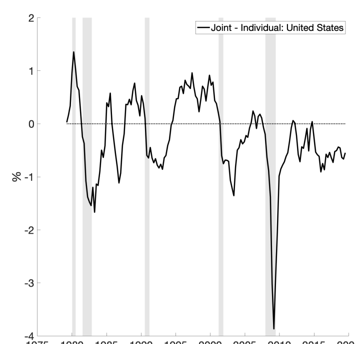
(d) Italy



(e) Japan



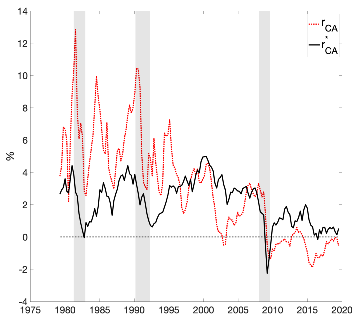
(f) United Kingdom



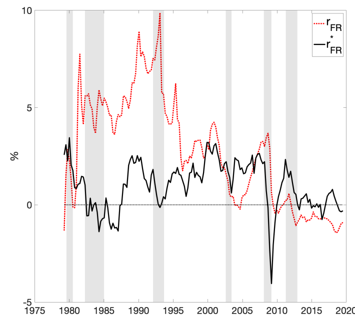
(g) United States

D.2 The Real Interest Rate vs The Natural Rate of Interest

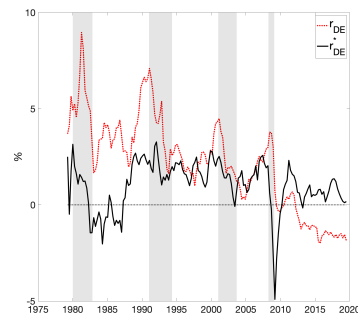
Figure 13: The Real Interest Rate vs The Natural Rate of Interest



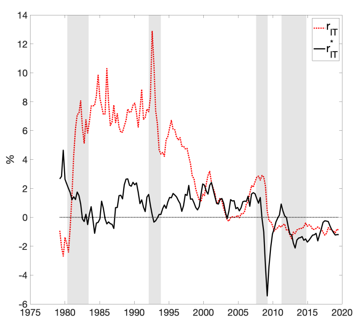
(a) Canada



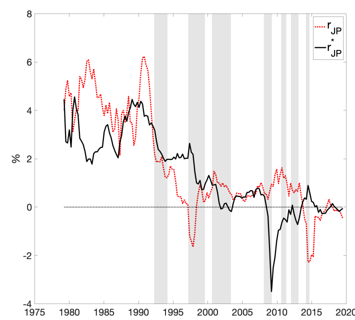
(b) France



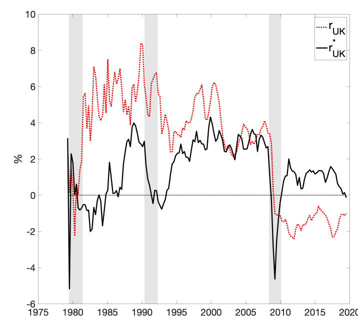
(c) Germany



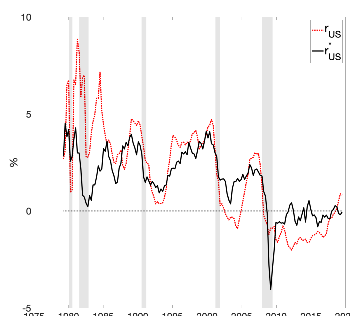
(d) Italy



(e) Japan



(f) United Kingdom



(g) United States

E Appendix: Tables

E.1 Posterior Estimates

Table 10: Posterior Estimates

Parameter	Canada	France	Germany	Italy	Japan	UK	US
$a_{y1,i}$	1.46	1.28	1.60	1.58	1.44	1.51	1.42
	(1.43, 1.48)	(1.26, 1.31)	(1.54, 1.63)	(1.54, 1.61)	(1.33, 1.48)	(1.49, 1.55)	(1.41, 1.44)
$a_{y2,i}$	-0.54	-0.44	-0.74	-0.74	-0.79	-0.68	-0.54
	(-0.56, -0.53)	(-0.47, -0.41)	(-0.79, -0.70)	(-0.77, -0.72)	(-0.84, -0.69)	(-0.71, -0.66)	(-0.56, -0.53)
$a_{r,i}$	-0.08	-0.05	-0.01	-0.06	-0.01	-0.03	-0.15
	(-0.10, -0.07)	(-0.06, -0.04)	(-0.02, -0.01)	(-0.07, -0.04)	(-0.02, -0.01)	(-0.04 , -0.02)	(-0.16, -0.12)
$b_{\pi,i}$	0.45	0.63	0.44	0.59	0.38	0.37	0.26
	(0.41, 0.47)	(0.58, 0.67)	(0.38, 0.48)	(0.56, 0.61)	(0.36, 0.40)	(0.35, 0.40)	(0.22, 0.29)
$b_{y,i}$	0.16	0.14	0.33	0.14	0.62	0.76	0.15
	(0.11, 0.20)	(0.13, 0.16)	(0.29, 0.40)	(0.08, 0.17)	(0.56, 0.66)	(0.71, 0.82)	(0.12, 0.16)
σ_{y_i}	0.47	0.18	0.18	0.16	0.03	0.25	0.21
	(0.43, 0.49)	(0.16, 0.19)	(0.17, 0.20)	(0.16, 0.18)	(0.02, 0.04)	(0.23, 0.26)	(0.20, 0.22)
σ_{π_i}	1.47	0.97	1.23	1.36	1.18	2.22	0.72
	(1.42, 1.49)	(0.96, 0.98)	(1.21, 1.29)	(1.34, 1.37)	(1.16, 1.23)	(2.21, 2.23)	(0.71, 0.74)
σ^*_i	0.33	0.26	0.82	0.50	0.93	0.47	0.49
	(0.32, 0.35)	(0.25, 0.27)	(0.80, 0.83)	(0.47, 0.51)	(0.91, 0.94)	(0.46, 0.48)	(0.48, 0.50)
σ_{g_i}	0.06	0.03	0.03	0.04	0.02	0.06	0.06
	(0.04, 0.06)	(0.02, 0.04)	(0.02, 0.04)	(0.03, 0.04)	(0.02, 0.03)	(0.06, 0.07)	(0.06, 0.06)
σ_{z_i}	0.20	0.20	0.30	0.07	0.23	0.09	0.09
	(0.18, 0.21)	(0.18, 0.24)	(0.29, 0.30)	(0.06, 0.09)	(0.21, 0.24)	(0.08, 0.10)	(0.08, 0.09)

$i = \{CA, FR, DE, IT, JP, UK, US\}$. In the parenthesis, the number on the left represents the 25th posterior draw and the number on the right represent the 75th posterior draw.

Table 11: Posterior Estimates: Common Parameters

Parameter	Density	Parameter1	Parameter2	Posterior Median
$\sigma_{g^{common}}$	Inverse-Gamma	0.03	0.03	0.05
				(0.05 , 0.05)
$\sigma_{z^{common}}$	Inverse-Gamma	0.1	0.1	0.12
				(0.10 , 0.17)
$\sigma_{g^{north-america}}$	Inverse-Gamma	0.03	0.03	0.06
				(0.06 , 0.07)
$\sigma_{z^{north-america}}$	Inverse-Gamma	0.1	0.1	0.17
				(0.16 , 0.18)
$\sigma_{g^{europe}}$	Inverse-Gamma	0.03	0.03	0.08
				(0.07 , 0.09)
$\sigma_{z^{europe}}$	Inverse-Gamma	0.1	0.1	0.26
				(0.25 , 0.27)

Parameter 1 is the mean of the normal distribution, the mean of the beta distribution, and the mode value of the inverse-gamma distribution. Parameter 2 is the standard deviation of the all of the distributions. MLE represents the estimates from the maximum likelihood on a country-by-country basis using the three-step estimation method in Holston et al. 2017. In the parenthesis, the number of the left represents the 25th posterior draw and the number of the right represent the 75th posterior draw.

E.2 Prior Specification for the Country-by-Country Estimation

Table 12: Prior Specification for the Country-by-Country Estimation

Parameter	Domain	Density	Parameter1	Parameter2
$a_{y1,i}$	$[0, \infty)$	Normal	MLE	0.3
$a_{y2,i}$	R	Normal	MLE	0.3
$a_{r,i}$	$(-\infty, -0.0025)$	Normal	min(MLE,-0.01)	0.3
$b_{\pi,i}$	[0,1]	Beta	MLE	0.3
$b_{y,i}$	(0.025, ∞)	Normal	min(MLE,0.5)	0.3
σ_{y_i}	(0,∞)	Inverse-Gamma	MLE	0.1
σ_{π_i}	(0,∞)	Inverse-Gamma	MLE	0.1
$\sigma_{y_i^*}$	(0,∞)	Inverse-Gamma	MLE	0.1
σ_{g_i}	(0,σ_{z_i})	Inverse-Gamma	0.03	0.03
σ_{z_i}	(0,∞)	Inverse-Gamma	0.1	0.1

$i = \{CA, FR, DE, IT, JP, UK, US\}$. Parameter 1 is the mean of the normal distribution, the mean of the beta distribution, and the mode value of the inverse-gamma distribution. Parameter 2 is the standard deviation of the all of the distributions.

E.3 Prior Specification for the Stationary Processes on the Preference Shifters

Table 13: Prior Specification for the Stationary Processes on the Preference Shifters

Parameter	Domain	Density	Parameter1	Parameter2
σ_{z_i}	(0,∞)	Inverse-Gamma	0.5	0.3
$\sigma_{z^{common}}$	(0, ∞)	Inverse-Gamma	0.5	0.3
$\sigma_{z^{north-america}}$	(0,∞)	Inverse-Gamma	0.5	0.3
$\sigma_{z^{europe}}$	(0,∞)	Inverse-Gamma	0.5	0.3
ρ_i	[0,1]	Beta	0.5	0.2
ρ^{common}	[0,1]	Beta	0.5	0.2
$\rho^{north-america}$	[0,1]	Beta	0.5	0.2
ρ^{europe}	[0,1]	Beta	0.5	0.2

$i = \{CA, FR, DE, IT, JP, UK, US\}$. ρ is the AR(1) coefficient. Parameter 1 is the mean of the normal distribution, the mean of the beta distribution, and the mode value of the inverse-gamma distribution. Parameter 2 is the standard deviation of the all of the distributions.